Interactions in nuclear matter

Agnieszka Sorensen
agnieszka.sorensen@gmail.com
Latest update: February 12, 2026

1. IDEAL FERMI GAS EXPRESSIONS

Previously, we obtained the following expressions for density, energy density, and pressure in the ideal relativistic Fermi gas at $T = 0$:

$$n = \frac{g p_F^3}{6\pi^2\hbar^3} \ , \tag{1}$$

$$\mathcal{E}_{\rm FG}\big|_{T=0} = \frac{g}{16\pi^2\hbar^3}\left[\frac{\epsilon_F^3 p_F}{c^2} + \epsilon_F p_F^3 - m^4c^5 \sinh^{-1}\left(\frac{p_F c}{mc^2}\right)\right] \tag{2}$$

$$= \frac{g}{16\pi^2\hbar^3}\left[\frac{\epsilon_F^3 p_F}{c^2} + \epsilon_F p_F^3 - m^4c^5 \ln\left(\frac{\epsilon_F + p_F c}{mc^2}\right)\right] \tag{3}$$

$$= \frac{g}{16\pi^2\hbar^3}\left[\frac{2\epsilon_F^3 p_F}{c^2} - \epsilon_F p_F m^2c^2 - m^4c^5 \ln\left(\frac{\epsilon_F + p_F c}{mc^2}\right)\right] \ , \tag{4}$$

$$P_{\rm FG}\big|_{T=0} = \frac{g}{16\pi^2\hbar^3}\left[\frac{2}{3}\epsilon_F p_F^3 - \epsilon_F p_F m^2c^2 + m^4c^5 \ln\left(\frac{\epsilon_F + p_F c}{mc^2}\right)\right] \ , \tag{5}$$

where g is the degeneracy, p_F is the Fermi momentum, m is the mass of the particle species in question, and $\epsilon_F = \sqrt{m^2 + p_F^2}$ is the (kinetic) Fermi energy. We have also obtained expressions for the case where T is finite:

$$n = \frac{g}{2\pi^2\hbar^3}\int_0^\infty dp\ p^2\ f_{\rm FD} \ , \tag{6}$$

$$\mathcal{E}_{\rm FG} = \frac{g}{2\pi^2\hbar^3}\int_0^\infty dp\ p^2\ \epsilon\ f_{\rm FD} \ , \tag{7}$$

and

$$P_{\rm FG} = \frac{gc^2}{6\pi^2\hbar^3}\int_0^\infty dp\ \frac{p^4}{\epsilon}\ f_{\rm FD} \ . \tag{8}$$

where $f_{\rm FD} = \left(e^{\beta(\varepsilon-\mu)} + 1\right)^{-1}$ is the Fermi-Dirac distribution function. Note that the expressions above assume that the system is spherically symmetric, i.e., uniform.

2. NUCLEAR MATTER: A FERMI GAS WITH INTERACTIONS

To model nuclear matter, we need to introduce interactions. With a known form of the interactions, a question one could then ask for any nuclear system at hand is "Given the form of the interactions and the positions and momenta of N nucleons, $(\boldsymbol{x}_1, \boldsymbol{p}_1, \boldsymbol{x}_2, \boldsymbol{p}_2, \ldots, \boldsymbol{x}_N, \boldsymbol{p}_N)$, what is the force on the i-th nucleon, $i = 1, \ldots, N$?" However, due to

1. the intrinsic non-perturbative nature of QCD (i.e., the impossibility of deriving elementary interactions between two nucleons from QCD), and

2. the complexity of modeling systems composed of many particles based on elementary particle-particle interactions (which means that even if one knows the force, solving for the properties of the system is impossible given the complexity of the mutual interactions between its many components, and which is often referred to as "the many-body problem"),

one needs to use a simplified approach. One of such simplified approaches is to ask: "Given a certain number of nucleons within a vicinity of a given point $\boldsymbol{x}$ (which is equivalent to a certain nucleon (or baryon) density n_B in the vicinity of $\boldsymbol{x}$), what is the average force on a nucleon at $\boldsymbol{x}$?"

It is more convenient to consider the problem in terms of a *potential* V that a particle feels at a given density n_B, rather than in terms of a force that a particle feels. In that case, in the presence of interactions one can say that the total energy of a particle is

$$\varepsilon = \sqrt{m^2 + p^2} + V(n_B) \ . \tag{9}$$

Consequently, also the energy density will now have a term corresponding to interactions,

$$\mathcal{E} = \mathcal{E}_{\rm FG} + \mathcal{E}_{\rm int} \ , \tag{10}$$

and similarly for the pressure

$$P = P_{\rm FG} + P_{\rm int} \ . \tag{11}$$

2.1. What is "nuclear matter"?

Before we introduce the form of V, $\mathcal{E}_{\rm int}$, and $P_{\rm int}$, let us first take a step back and establish what one really means by "nuclear matter".

First, let's discuss some basic properties of nuclei, which will inform our expectations for the behavior of nuclear matter. Scattering experiments and phenomenology [1] show that the volume of a nucleus is approximately a linear function of the number of nucleons N,

$$V \approx \alpha N \ , \tag{12}$$

where α is some proportionality constant. Note that this means that $dN = dV/\alpha = (4\pi r^2 \ dr)/\alpha$ (where we assumed that the nucleus is spherically symmetric), and so the number of nucleons ΔN within any infinitesimal shell of the nucleus of width ΔR is

$$\Delta N = \frac{4\pi}{\alpha} \int_R^{R+\Delta R} dr \ r^2 = \frac{4\pi}{\alpha} \left[\frac{(R+\Delta R)^3}{3} - \frac{R^3}{3} \right] = \frac{\Delta V}{\alpha} \ , \tag{13}$$

where ΔV is the volume of the shell. Consequently, the number density of nucleons in any such shell is

$$n = \frac{\Delta N}{\Delta V} = \frac{1}{\alpha} \ , \tag{14}$$

a constant. That is, the density of nucleons in a nucleus is approximately the same at any radius and for all nuclei. If one looks at this problems a little closer, it turns out that there are additional surface effects (that is effects due to the fact that a nucleus has a finite size and a surface) that one needs to take into account, as well as effects due to the Coulomb interaction or an asymmetry in the number of protons and neutrons. Nevertheless, it still follows that the central densities of all nuclei are similar [1]. (Note that this is different than in the case of long range forces, such as for example gravity, which causes more massive stars to have denser cores.)

The above in turn implies that the range of the strong interaction is small, so that while adjacent nucleons are very strongly bound to each other, their effect on nucleons in a more distant part of the nucleus is much smaller (if it were not the case, then again the density would not be approximately constant throughout the nucleus). Additionally, it means that the nuclear forces become repulsive at very small separations (further preventing more massive nuclei from forming denser cores), and as a consequence the nuclear binding forces are the largest (saturate) at some close distance r_0. Indeed, experiments show that the nucleon-nucleon interaction exhibits a strong attraction for an internucleonic distance of about 1.4 fm, while for smaller distances the potential sharply rises and becomes strongly

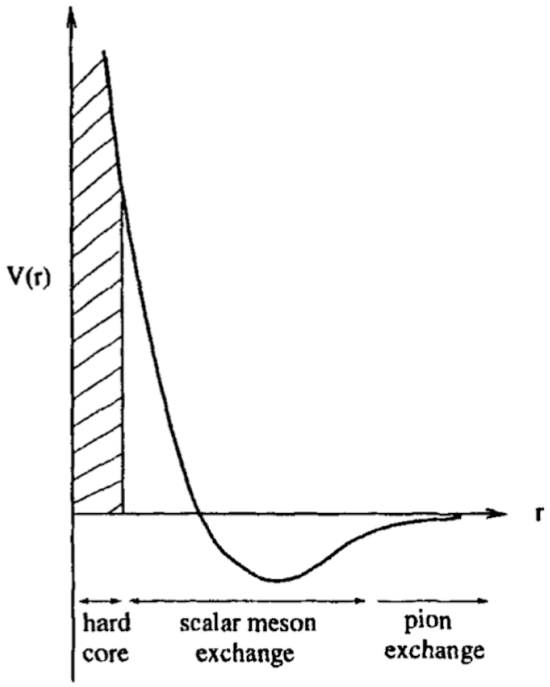

FIG. 1: A sketch of the nucleon-nucleon potential as a function of distance r between two nucleons [1]. The hard core radius is around 0.4 fm, the attraction at intermediate ranges if to be found at radii ≈ 1 fm, and the long-range attraction starts at around 2 fm.

repulsive at about 0.5 fm, see Fig. 1. This property of the nuclear force is often referred to as *saturation of nuclear matter*, and it means that there is a certain density of nucleons n_0 (corresponding to some average distance r_0 between the nucleons) at which the energy of the system is minimized (i.e., binding forces are the largest).

Because exact calculations involving finite nuclei are a formidable task even when using effective approaches, one often considers a simplified scenario: *nuclear matter*, that is a uniform, infinite, isospin-symmetric (composed of an equal number of protons and neutrons) system in which the Coulomb interaction is neglected. In this limit, nuclear matter is well-described by an interacting Fermi gas at zero temperature. By definition, the system is translationally invariant and therefore must be composed out of the eigenstates of the momentum operator, which means that the single-particle states are plane waves, just as in the case of the ideal Fermi gas. Moreover, this means that the density of the momentum states in the two cases (ideal and interacting) is identical, so that Eqs. (1) and (6) still apply in the interacting case, and the ideal contributions to both energy density and pressure are also the same. Having in this way sidestepped the need for calculating the wave-functions describing the states of the system (a task central to problems involving finite nuclei, in analogy to, e.g., calculations of the wave functions of a hydrogen atom), the core quantity of interest in this approach is the energy per particle E/N (where E denotes the total energy and N denotes the total number of nucleons) as a function of baryon (nucleon) density n_B.

If one then considers a uniform many-body system composed of nucleons, the energy per particle must exhibit a minimum occurring at the equilibrium density n_0 and characterized by some value B_0, $B = E(n_0)/N - m_N = B_0$ (here, we subtract the trivial contribution to the energy per particle due to the nucleon mass m_N; in result, B is nothing else than the binding energy), see Fig. 2. The values of parameters B_0 and n_0 can be established based on the experimentally observed properties of nuclei, for example as encoded in the semi-empirical Bethe-Weizsäcker mass formula [2] extrapolated to the case of infinite nuclear matter, and are approximately equal $B_0 \approx -16$ MeV and $n_0 \approx 0.160$ fm^{-3} [4].

In view of these universal properties of nuclei, any model claiming to describe nuclear matter must reproduce its saturation properties encoded in the values of B_0 and n_0. Interestingly, two different potentials that lead to the same scattering properties of nuclei (which can be compared with results of scattering experiments) may at the same time yield disparate results for the properties of nuclear matter; naturally, a potential implying unphysical properties of nuclear matter should be abandoned. For example, it is known that nuclear potentials based on even very sophisticated two-body forces, fit to scattering experiments, do not satisfactorily reproduce the saturation properties of nuclear matter, while at the same time, inclusion of somewhat simplistic three-body forces results in a much better

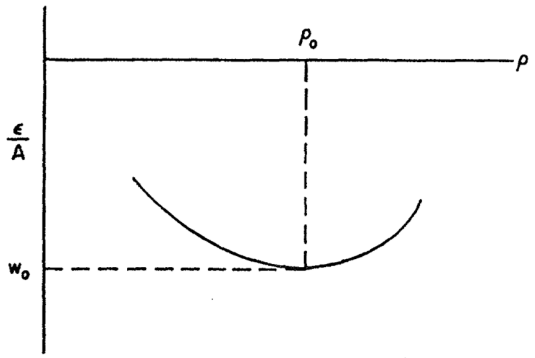

FIG. 2: A sketch of the energy per particle in nuclear matter [3]. In this figure, energy density is denoted as ϵ, the number of nucleons as A, the nucleon density as ρ, the saturation density as ρ_0, and the binding energy at saturation as w_0.

agreement with data [4].

On the other hand, one can also reverse the problem and fit the parameters of a given potential not to scattering data, but such that the saturation properties of nuclear matter are reproduced. This is the approach taken here.

2.2. Nuclear matter interactions

Because we want the nuclear matter to exhibit a certain behavior as a function of density n_B, it is useful to parametrize the interactions as a function of density. A minimal model describing a potential that is attractive at large distances (small densities) and repulsive at small distances (large densities), such as shown in Fig. 1, has two terms: an attractive term and a repulsive term. Consider then a single-particle potential of the form

$$V(n_B) = A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau-1} . \tag{15}$$

Here, n_0 is the saturation density, while A, B, and τ are *parameters* of the potential that are fit to reproduce chosen properties of nuclear matter.

Given Eq. (15), one can calculate the total contribution to the energy density due to the interactions,

$$\mathcal{E}_{\rm int}(n_B) = \int dn_B\ V(n_B) = \frac{A}{2}\frac{n_B^2}{n_0} + \frac{B}{\tau}\frac{n_B^\tau}{n_0^{\tau-1}} . \tag{16}$$

One can show that the above result is true using a somewhat more advanced formalism of *functional derivatives of the energy density with respect to the nucleon distribution function* (see, e.g., [5, 6]). Here, let me motivate the above in the following way: At $T = 0$, the chemical potential of the system μ_B is equal to the Fermi energy of the system, which in the presence of interactions is given by

$$\varepsilon_F = \sqrt{m^2 + p_F^2} + V_{\rm int} = \epsilon_F + V_{\rm int} = \mu_B(n_B) , \tag{17}$$

where we denote the kinetic contribution to the Fermi energy as $\epsilon_F = \sqrt{m^2 + p_F^2}$. At the same time, from the definition, $\mu_B = d\mathcal{E}/dn_B$, and consequently

$$\mathcal{E}(n_B) = \int_0^{n_B} dn'\ \mu_B(n') = \int_0^{n_B} dn'\ \epsilon_F + \int_0^{n_B} dn'\ V_{\rm int} . \tag{18}$$

The first term in the above equation will integrate to the ideal contribution to the energy density, $\mathcal{E}_{\rm FG}$, while the second term integrates to the contribution due to the interactions, Eq. (16). Now, since the interaction potential, Eq. (15), depends only on density and *not* on temperature, its contribution to μ_B will be the same regardless of the fact whether $T = 0$ or $T > 0$. That is, while $d\mathcal{E}_{\rm FG}/dn_B$ depends on the temperature, $d\mathcal{E}_{\rm int}/dn_B$ does not. As a consequence, Eq. (16), easily derived in the case of $T = 0$, is actually valid at any value of T.

We assume a similar strategy to obtain the expression for the pressure. It can be shown that the pressure at $T = 0$ is given in terms of energy per particle as,

$$P\big|_{T=0} = n_B^2 \frac{d\left(\frac{\mathcal{E}}{n_B}\right)}{dn_B} \ . \tag{19}$$

Similarly as above, we can decompose the total energy density into the ideal and interaction part, $\mathcal{E} = \mathcal{E}_{\rm FG} + \mathcal{E}_{\rm int}$, so that

$$P\big|_{T=0} = n_B^2 \frac{d\left(\frac{\mathcal{E}_{\rm FG}}{n_B}\right)}{dn_B} + n_B^2 \frac{d\left(\frac{\mathcal{E}_{\rm int}}{n_B}\right)}{dn_B} \ . \tag{20}$$

Again, the first term in the expression above will just amount to the ideal contribution to the pressure, $P_{\rm FG}$. The second term can be explicitly calculated to yield

$$P_{\rm int}(n_B) = n_B^2 \frac{d}{dn_B}\left(\frac{A}{2}\frac{n_B}{n_0} + \frac{B}{\tau}\frac{n_B^{\tau-1}}{n_0^{\tau-1}}\right) = \frac{A}{2}\frac{n_B^2}{n_0} + B\frac{\tau-1}{\tau}\frac{n_B^{\tau}}{n_0^{\tau-1}} \ . \tag{21}$$

Now, by similar arguments as above, one can see that this contribution to the pressure due to the interactions will remain the same whether we're considering a system at $T = 0$ or at $T > 0$.

Altogether, we obtain a general expression for both the energy density and pressure in a system where interactions have the form given in Eq. (15),

$$\mathcal{E}(n_B) = \mathcal{E}_{\rm FG} + \frac{A}{2}\frac{n_B^2}{n_0} + \frac{B}{\tau}\frac{n_B^{\tau}}{n_0^{\tau-1}} \ , \tag{22}$$

$$P(n_B) = P_{\rm FG} + \frac{A}{2}\frac{n_B^2}{n_0} + B\frac{\tau-1}{\tau}\frac{n_B^{\tau}}{n_0^{\tau-1}} \ . \tag{23}$$

2.3. Fitting the parameters of the potential

Now we want to fix the values of the parameters A, B, and τ such that our simple model reproduces the known properties of nuclear matter. We described two of these properties above: the binding energy $B_0 \approx -16$ MeV and the saturation density $n_0 \approx 0.160$ fm^{-3}.

The condition for the binding energy amounts to demanding that the parameters A, B, and τ are such that the following equation,

$$\frac{\mathcal{E}}{n_B}\bigg|_{n_B=n_0} - m_N = B_0 \ , \tag{24}$$

is satisfied. Note that explicitly, the above condition can be rewritten as

$$\frac{\mathcal{E}_{\rm FG}}{n_0} + \left(\frac{A}{2} + \frac{B}{\tau}\right) - m_N = B_0 \ . \tag{25}$$

The condition requiring that the minimum of the binding energy occurs at $n_B = n_0$ can be expressed as a condition for the extremum of $\mathcal{E}/n_B$,

$$\frac{d}{dn_B}\left(\frac{\mathcal{E}}{n_B} - m_N\right)\bigg|_{n_B=n_0} = \frac{d}{dn_B}\left(\frac{\mathcal{E}}{n_B}\right)\bigg|_{n_B=n_0} = 0 \ . \tag{26}$$

As mentioned above, it can be shown that

$$\frac{d\mathcal{E}_{\rm FG}}{dn_B} = \epsilon_F \ , \tag{27}$$

so that

$$\frac{d}{dn_B}\left(\frac{\mathcal{E}_{\rm FG}}{n_B}\right)\bigg|_{n_B=n_0} = \frac{\epsilon_F(n_0)}{n_0} - \frac{\mathcal{E}_{\rm FG}(n_0)}{n_0^2} \ , \tag{28}$$

while

$$\frac{d}{dn_B}\left(\frac{\mathcal{E}_{\rm int}}{n_B}\right)\bigg|_{n_B=n_0} = \frac{A}{2}\frac{1}{n_0} + B\frac{\tau-1}{\tau}\frac{1}{n_0}\ , \tag{29}$$

so that altogether the condition for the minimum can be explicitly written as

$$\epsilon_F^{(\rm kin)}(n_0) - \frac{\mathcal{E}_{\rm FG}(n_0)}{n_0} + \frac{A}{2} + B\frac{\tau-1}{\tau} = 0\ . \tag{30}$$

Since we have three parameters to fix, we need a third equation. There is another property of nuclear matter that we haven't discussed yet, known is the *incompressibility at saturation density* K_0. Incompressibility K is a measure of how difficult it is to increase the density of a given system. If the incompressibility is large, then the system is highly incompressible, and therefore it is hard to increase its density. In general, incompressibility is a function of density: if, for example, interactions in the system are more repulsive as the density increases, then it is harder to compress the system at higher densities, compared to the effort required at lower densities. Incompressibility at saturation density K_0 is, therefore, a measure of how hard it is to compress nuclear matter as found in the cores of heavy nuclei. Note that if the interactions are more repulsive, it means that the pressure is higher, and *vice versa*. Therefore, incompressibility can be defined as

$$K \equiv 9\frac{dP}{dn_B}\ . \tag{31}$$

The factor of 9 in the equation above is a historical artifact, stemming from the definition of incompressibility at zero temperature and saturation density, $n_B = n_0$, adopted in the early days,

$$K_0 \equiv \left(p_F^2\frac{d^2}{dp_F^2}\left(\frac{\mathcal{E}^{(T=0)}}{n_B}\right)\right)_{n_B=n_0}\ . \tag{32}$$

Experimentally, incompressibility is measured by inducing monopole (radial) collective oscillations in heavy nuclei (that is, inducing the density within the entire nucleus to oscillate radially so that its radius oscillates as well), also known as "breathing modes" (since the radius of the nucleus is oscillating, it "looks" like the nucleus is breathing). Since all the nucleons within the nucleus are oscillating together, these oscillations can also be viewed as being resonant, and hence the name most often used to describe them is "the giant monopole resonance" (GMR). To measure the incompressibility using the GMR, one typically studies the excitation energy of the GMR in heavy nuclei. The GMR is sensitive to the incompressibility of nuclear matter because the incompressibility of nuclear matter is related to the frequency (or, equivalently, energy) of the GMR mode. A higher incompressibility corresponds to a higher excitation energy for the GMR. Experimental techniques involving scattering of nucleons or alpha particles are used to excite the GMR and measure its energy. This is then compared with theoretical calculations, which can provide predictions for the GMR energy based on the assumed incompressibility value.

The measurement is difficult, and the comparison to theoretical calculations is model-dependent. Therefore, the value of K_0 is only known within some bounds. For example, the analysis performed in [7] establishes that $K_0 = 231 \pm 5$ MeV, the work presented in [8] leads to the value of $K_0 = 215 \pm 5$ MeV, while in [9] the range of K_0 between 240 and 270 MeV is obtained. In fact, changing the value of incompressibility at saturation density across a vast range of values is how a variation in the equation of state used to be obtained in simulations of heavy-ion collisions (nowadays, there are more sophisticated equations of state available, so that the value of K_0 can be fixed within the experimentally measured bounds while still allowing for a nontrivial variation in the density-dependence of the equation of state [6]).

With a chosen value of K_0, the last condition needed to fix all three parameters of the nuclear matter interactions follows immediately from Eq. (31),

$$9\frac{dP}{dn_B}\bigg|_{n_B=n_0} - K_0 = 0\ . \tag{33}$$

All three conditions in Eqs. (24), (26), and (33) must be satisfied simultaneously to obtain the correct values of A, B, and τ for a given set of chosen nuclear matter properties B_0, n_0, and K_0. This means that one needs to solve a 3-dimensional root problem to obtain A, B, and τ.

2.4. Particular values of the parameters of the potential

There are known parametrizations of the potential parameters in the literature. A particular parametrization based on the model developed in [6], leading to $n_0 = 0.160$ fm^{-3}, $B_0 = -16.3$ MeV, and $K_0 = 240$ MeV, is

$$A = -218.699 \text{ MeV} , \quad \text{B} = 166.215 \text{ MeV} , \quad \tau = 2.33376 . \tag{34}$$

Another parametrization, corresponding to a "harder" equation of state with the same values of n_0 and B_0 but with $K_0 = 300$ MeV, is

$$A = -153.658 \text{ MeV} , \quad \text{B} = 101.174 \text{ MeV} , \quad \tau = 2.61421 . \tag{35}$$

3. CALCULATING THERMODYNAMIC OBSERVABLES AT FINITE T

To calculate the energy density or pressure of interacting nuclear matter, Eqs. (22) and (23), one needs to calculate the kinetic energy contribution to the energy density $\mathcal{E}_{\rm FG}$ or pressure $P_{\rm FG}$. Note that these carry the subscript "FG", suggesting that they are the energy density and pressure of an ideal (non-interacting) Fermi gas, given by Eqs. (7) and (8). The form of the integrals is indeed the same as in the case of an ideal Fermi gas, however, now the value of ε entering the Fermi-Dirac distribution function is the total energy of a particle, including its potential energy.

In order to calculate any integral over the Fermi distribution, one needs to know the value of the temperature T and the chemical potential μ_B which enter the expression for the Fermi-Dirac distribution. However, in many applications the chosen primary thermodynamic variables (that is those that one freely varies) are not T and μ_B, but T and n_B. Therefore, the problem at hand is to obtain μ_B given T and n_B. This can be done by utilizing Eq. (6), repeated here with the Fermi-Dirac function written out explicitly:

$$n_B = \frac{g}{2\pi^2\hbar^3} \int_0^\infty dp\ p^2\ \frac{1}{e^{\beta(\varepsilon-\mu_B)}+1} . \tag{36}$$

We know the values of n_B and T, and therefore we might want to invert the above equation to obtain μ_B. However, this cannot be done analytically. Nevertheless, one can find the value of μ_B which for a given T reproduces the given n_B numerically by solving for the roots of the following equation,

$$n_B - \frac{g}{2\pi^2\hbar^3} \int_0^\infty dp\ p^2\ \frac{1}{e^{\beta(\varepsilon-\mu_B)}+1} = 0 . \tag{37}$$

Before we do that, let us investigate the behavior of μ_B as a function of n_B so that we know what to expect when solving this problem. In order to do so, let us consider the case where the temperature T is finite, but still very close to 0. In that case, one can expect that the value of the chemical potential is close to the value of the Fermi energy, see Eq. (17),

$$\mu_B \approx \epsilon_F + V_{\rm int} = \epsilon_F + A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau-1} . \tag{38}$$

For given values of the potential parameters it is then easy to plot μ_B as a function of n_B. In Fig. 3, the dependence of μ_B on n_B is shown at $T = 0$ for both the ideal Fermi gas and for nuclear matter with potential parameters as specified in Eq. (34). Note that while the chemical potential of the ideal Fermi gas is a monotonic function of n_B, in the case of nuclear matter the chemical potential has a polynomial-like behavior, i.e., for certain values of μ_B there are multiple values of n_B leading to that μ_B (this is, of course, not a surprise, as the interaction term is a polynomial). This is why, in approaches where T and μ_B are the primary variables, calculating n_B given T and μ_B can be problematic: for every (T, μ_B) there may be up to three possible answers! This is not the case when one wants to calculate μ_B based on given T and n_B: for every (T, n_B) there is only one possible answer.

It turns out that one can further simplify the problem. Explicitly, for a system with interactions, the Fermi-Dirac distribution function is

$$f_{\rm FD} = \frac{1}{e^{\beta(\varepsilon-\mu_B)}+1} = \frac{1}{e^{\beta(\epsilon+V_{\rm int}-\mu_B)}+1} . \tag{39}$$

Note that one can always define a new variable, known as the *effective chemical potential*,

$$\mu^* = \mu_B - V_{\rm int} , \tag{40}$$

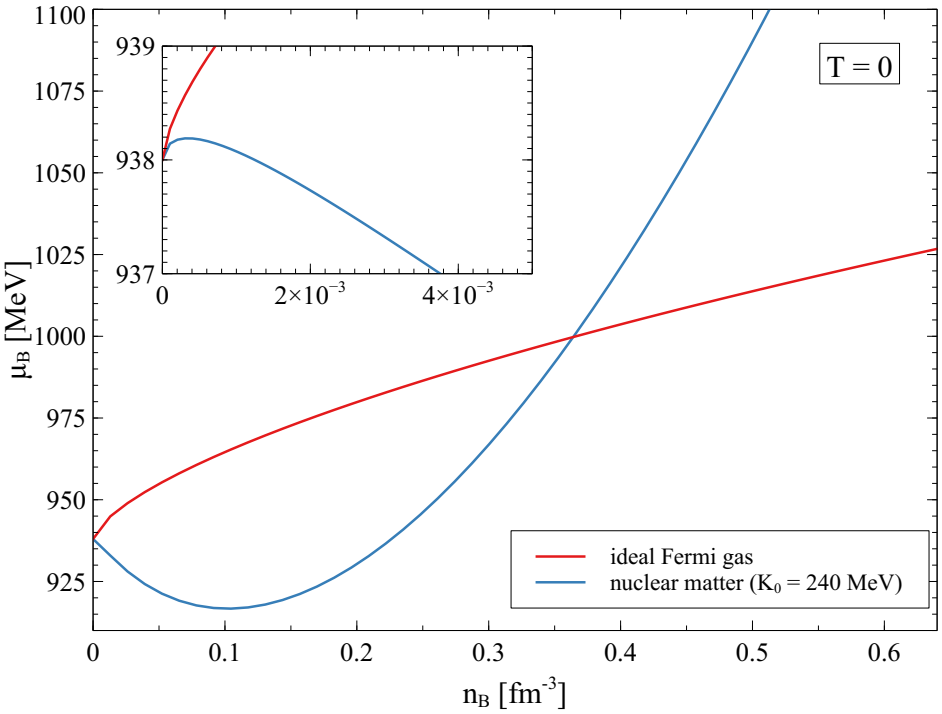

FIG. 3: The dependence of the chemical potential μ_B on the number density n_B, at $T = 0$, for an ideal Fermi gas and for nuclear matter with potential parameters as specified in Eq. (34).

in terms of which the Fermi distribution becomes

$$f_{\rm FD} = \frac{1}{e^{\beta(\varepsilon-\mu_B)}+1} = \frac{1}{e^{\beta(\epsilon-\mu^*)}+1} \ . \tag{41}$$

With such form, the expression for the number density, Eq. (36), looks just like the one for ideal Fermi gas. In fact, the same can be done in the case of the expressions for the energy density and pressure. As a result, these expressions, written as a function of T and μ^*, look *exactly* like those for an ideal Fermi gas. Therefore, for calculating the energy density and pressure of interacting nuclear matter given T and n_B, it is enough to solve for the effective chemical potential μ^* using the appropriately modified root equation Eq. (37), then use that value of μ^* in place of μ_B in expressions for the ideal (non-interacting) Fermi gas energy density and pressure, and add to these the interaction terms in order to obtain the energy density and pressure of interacting nuclear matter. That is, for the purposes of calculating $\mathcal{E}$ and P, one doesn't need to know what the real chemical potential μ_B is and one can just reuse the expressions for the ideal Fermi gas at $T > 0$. In case the information about the "real" chemical potential is needed, one can easily obtain μ_B from μ^* by inverting Eq. (40).

Finally, let us acknowledge that the integral in Eq. (37) has the upper limit at infinity. This, of course, is not ideal for numerical applications. Luckily, such integrals can be easily rewritten in terms of integral over the interval $(0, 1)$ by using a simple substitution of variables,

$$p = \frac{1-t}{t} \ . \tag{42}$$

With this, we have $dp = -dt/t^2$ and $t = 1/(p+1)$, so that

$$n_B = \frac{g}{2\pi^2\hbar^3}\int_{t(p=0)}^{t(p=\infty)} \left(-\frac{dt}{t^2}\right) \left(\frac{1-t}{t}\right)^2 \frac{1}{e^{\beta\left(\sqrt{m^2+\left(\frac{1-t}{t}\right)^2}-\mu^*\right)}+1} \tag{43}$$

$$= \frac{g}{2\pi^2\hbar^3}\int_0^1 dt\ \frac{(1-t)^2}{t^4} \frac{1}{e^{\beta\left(\sqrt{m^2+\left(\frac{1-t}{t}\right)^2}-\mu^*\right)}+1} \ , \tag{44}$$

which is easy to integrate numerically.

4. EXERCISES

1. Show that the condition for the minimum of the binding energy at $n_B = n_0$ and $T = 0$, Eq. (26), is equivalent to demanding that the pressure satisfies $P(n_B = n_0, T = 0) = 0$.

2. Show that Eqs. (31) and (32) are equivalent.

3. Using values of the parameters provided in Eqs. (34) and (35), calculate what is the potential felt by a single nucleon at saturation density.

---

[1] S. M. Wong, "Introductory Nuclear Physics," John Wiley & Sons, Ltd. (1998). 2, 3

[2] Wikipedia article on the Bethe-Weizsäcker formula, URL: https://en.wikipedia.org/wiki/Semi-empirical_mass_formula, accessed: July 7, 2023. 3

[3] B. D. Day, Rev. Mod. Phys. **39**, 719-744 (1967) doi:10.1103/RevModPhys.39.719 4

[4] H. A. Bethe, "Theory of nuclear matter," Ann. Rev. Nucl. Part. Sci. **21**, 93-244 (1971) doi:10.1146/annurev.ns.21.120171.000521. 3, 4

[5] G. Baym and S. A. Chin, Nucl. Phys. A **262**, 527-538 (1976) doi:10.1016/0375-9474(76)90513-3 4

[6] A. Sorensen and V. Koch, "Phase transitions and critical behavior in hadronic transport with a relativistic density functional equation of state," Phys. Rev. C **104**, no.3, 034904 (2021) doi:10.1103/PhysRevC.104.034904 arXiv:2011.06635 [nucl-th]. 4, 6, 7

[7] D. H. Youngblood, H. L. Clark and Y. W. Lui, Phys. Rev. Lett. **82**, 691-694 (1999) doi:10.1103/PhysRevLett.82.691 6

[8] J. P. Blaizot, J. F. Berger, J. Decharge and M. Girod, Nucl. Phys. A **591**, 435-457 (1995) doi:10.1016/0375-9474(95)00294-B 6

[9] D. T. Khoa, G. R. Satchler and W. von Oertzen, Phys. Rev. C **56**, 954-969 (1997) doi:10.1103/PhysRevC.56.954 6