The QCD Phase Diagram and Beam Energy Scan Physics: A Theory Overview

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FRIB

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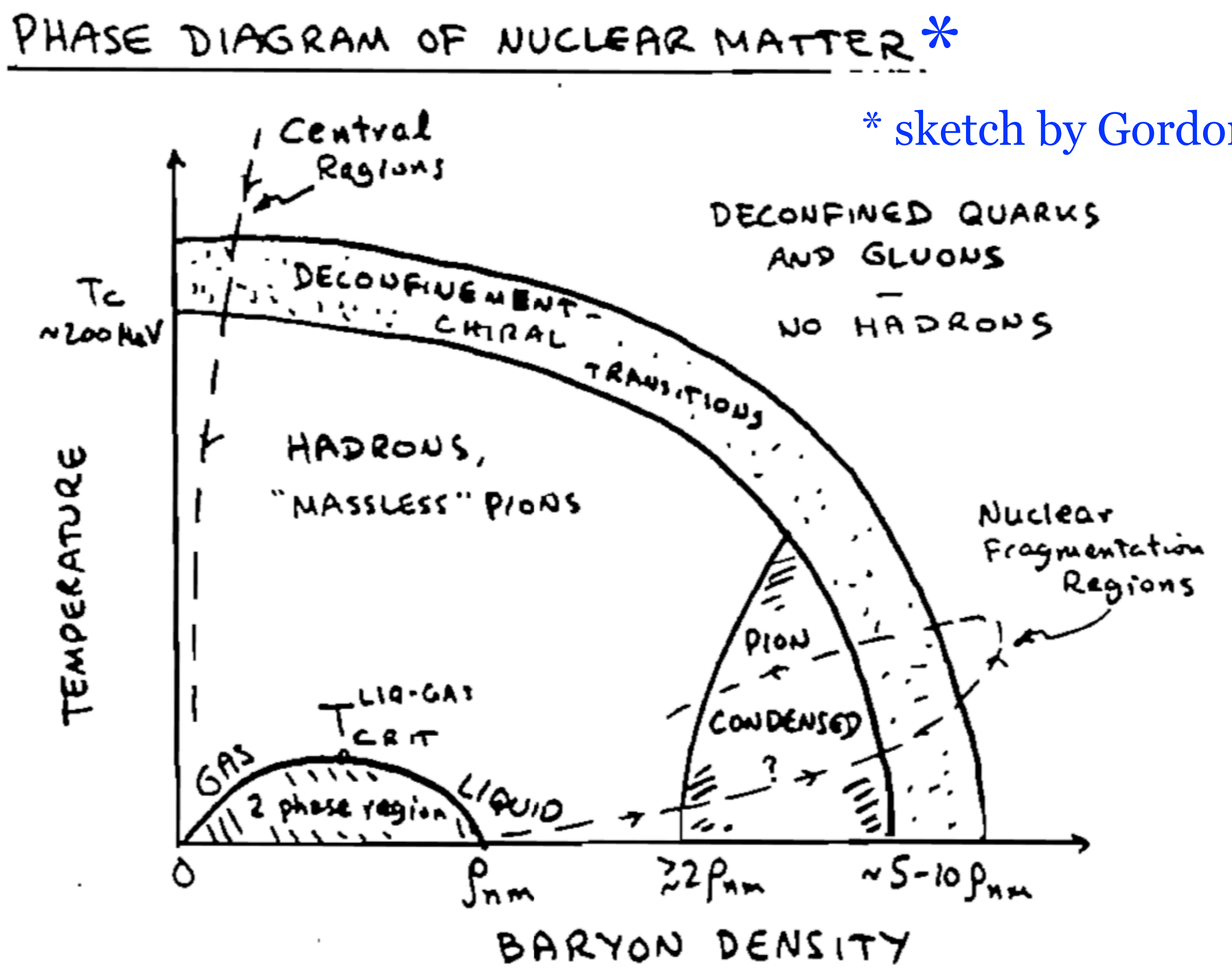
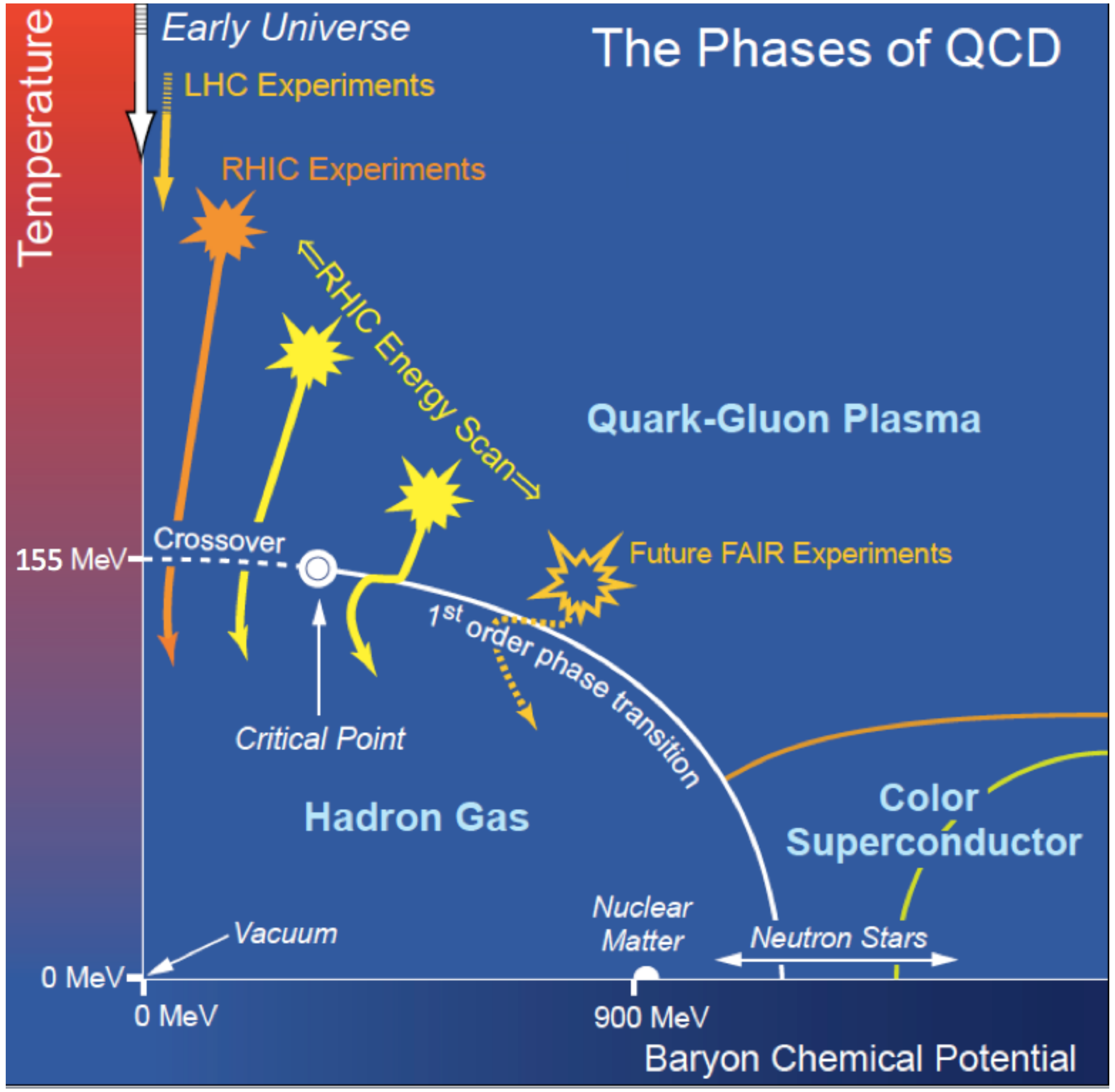
2024 Fall Meeting of the APS Division of Nuclear Physics

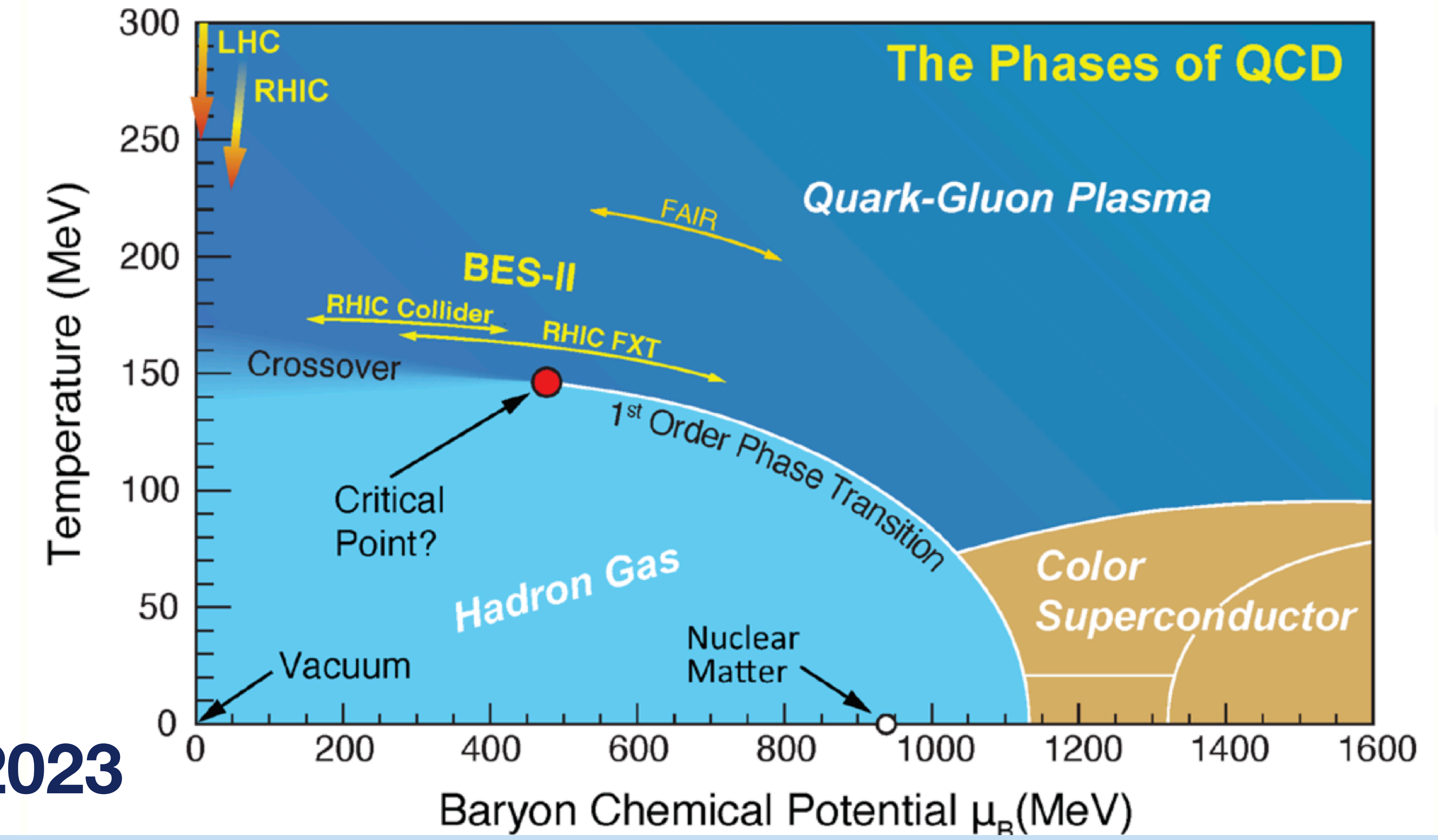
The idea to probe the QCD phase diagram with heavy-ions is not new…



* sketch by Gordon Baym, LRP 1983



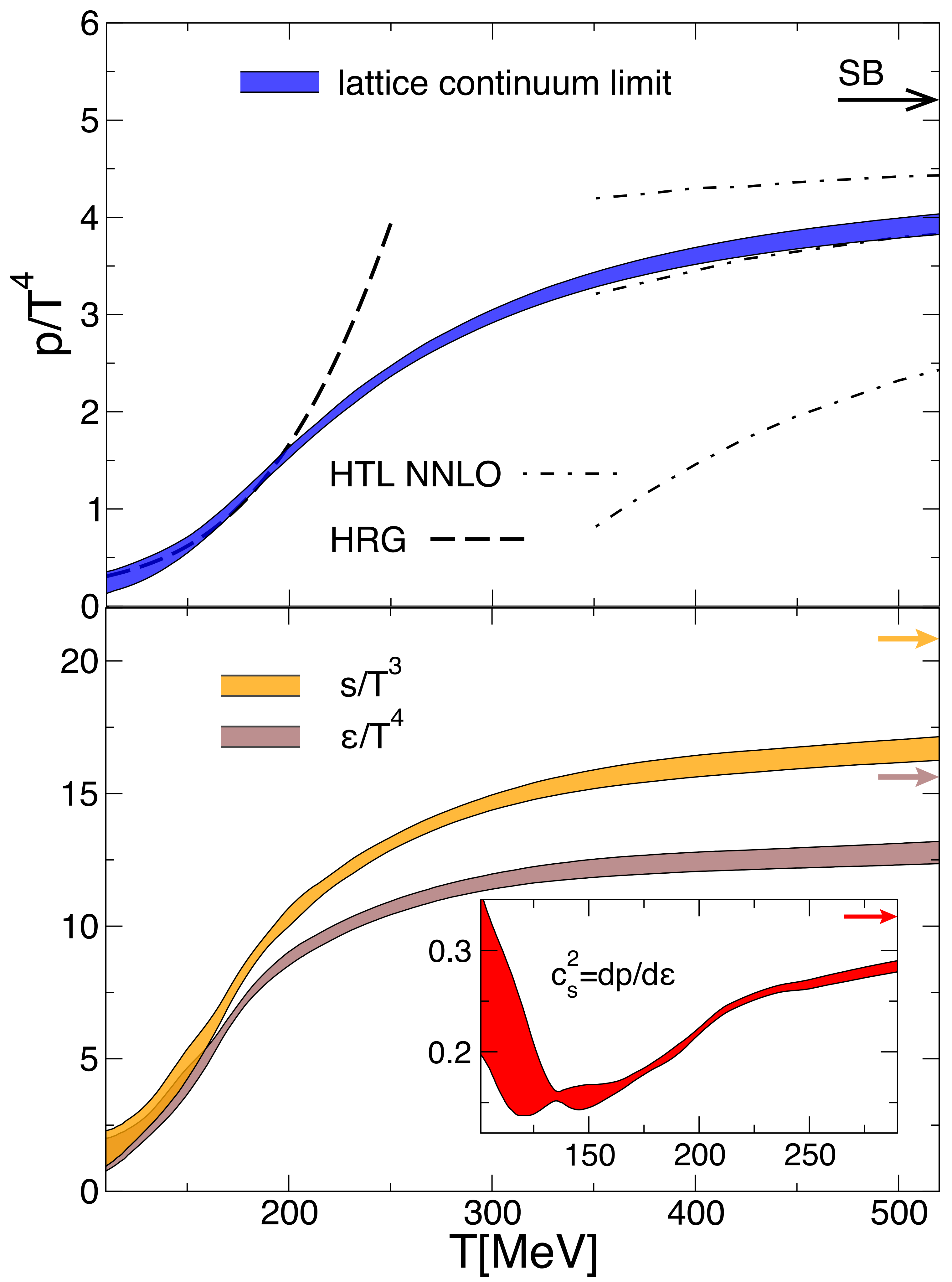
LRP 2007

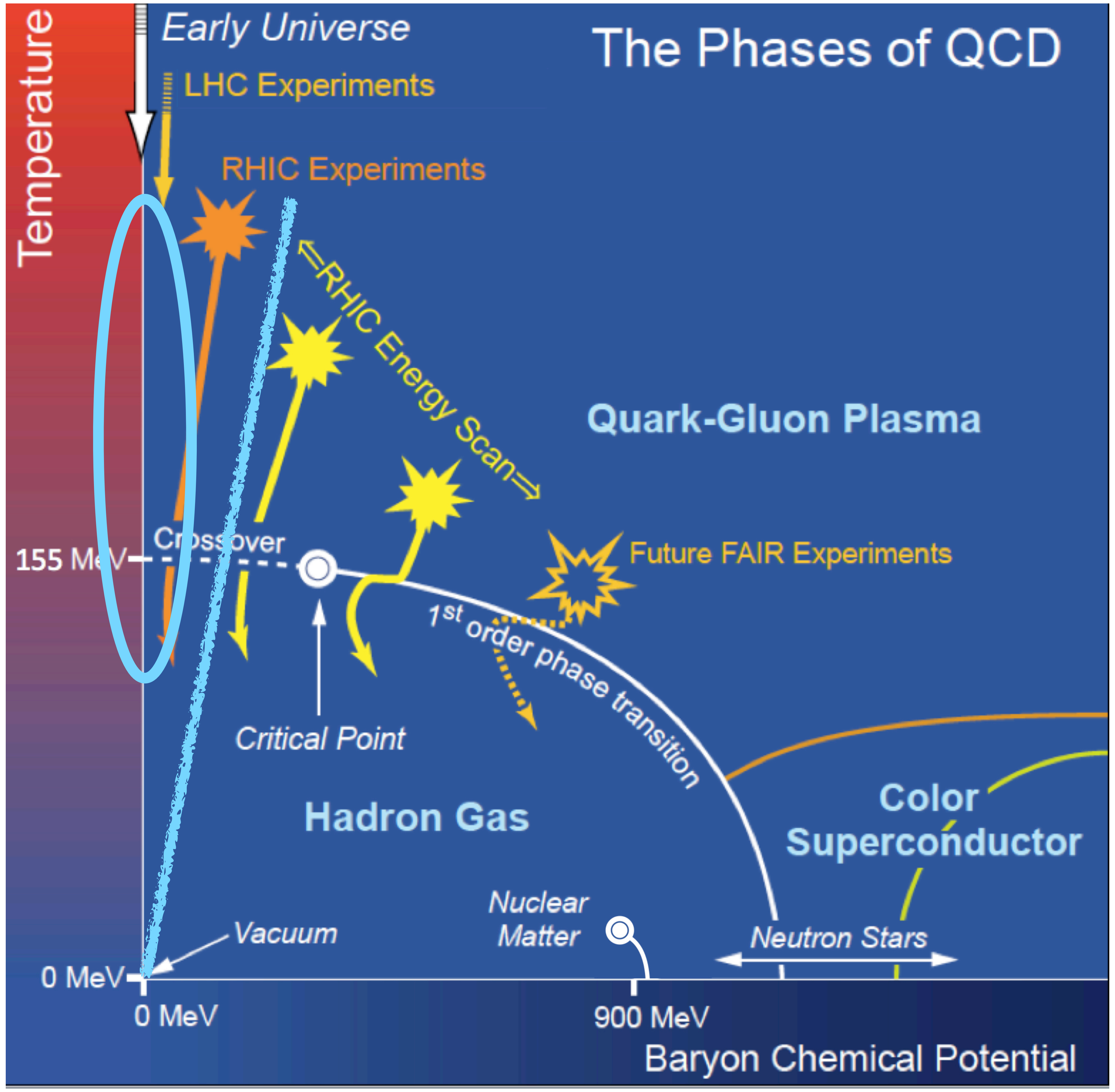


LRP 2023

The QCD phase diagram: what do we know?

LQCD EOS: $T_{pc}(\mu_B = 0) \approx 155\ \mathrm{MeV}$

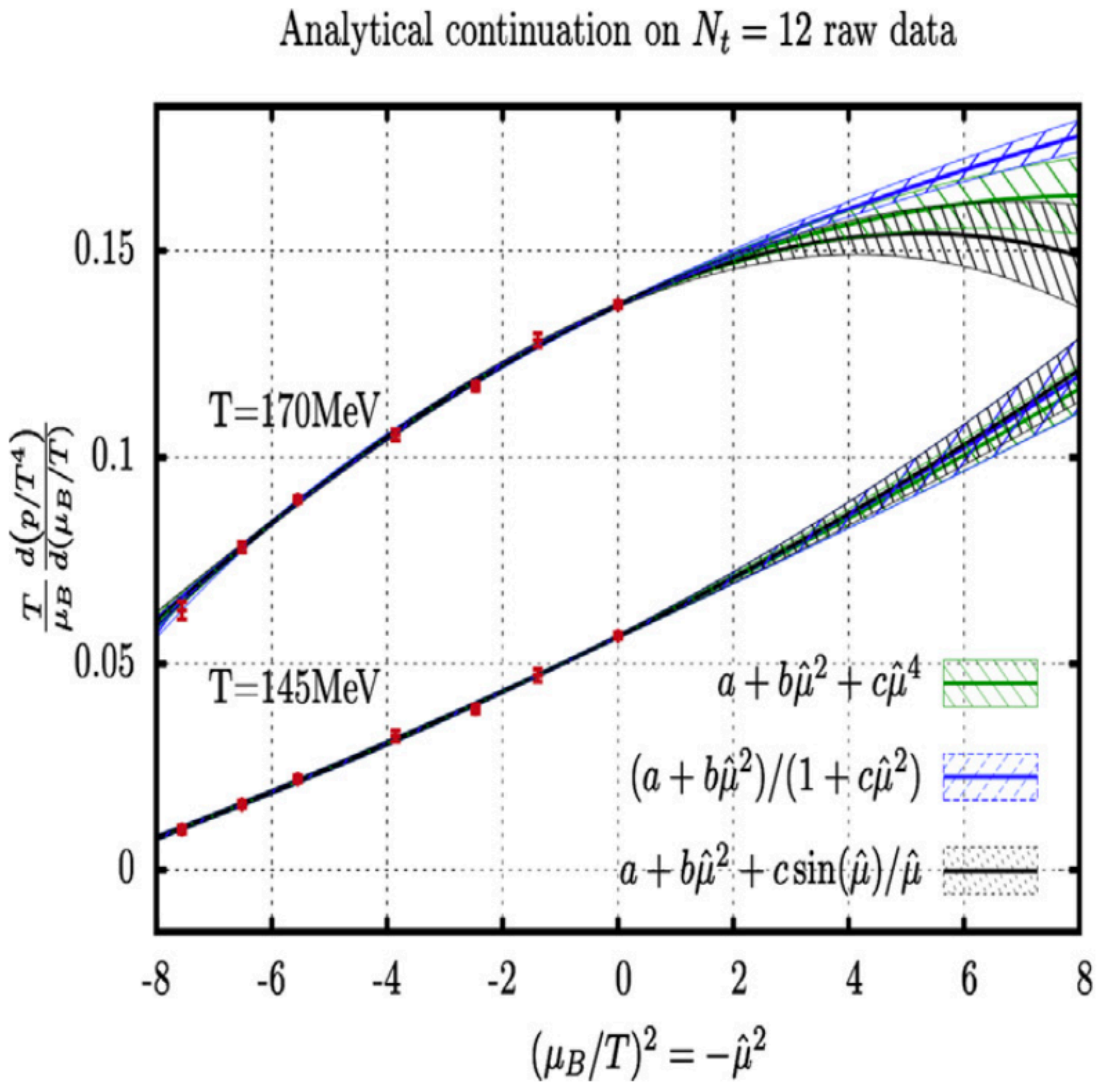


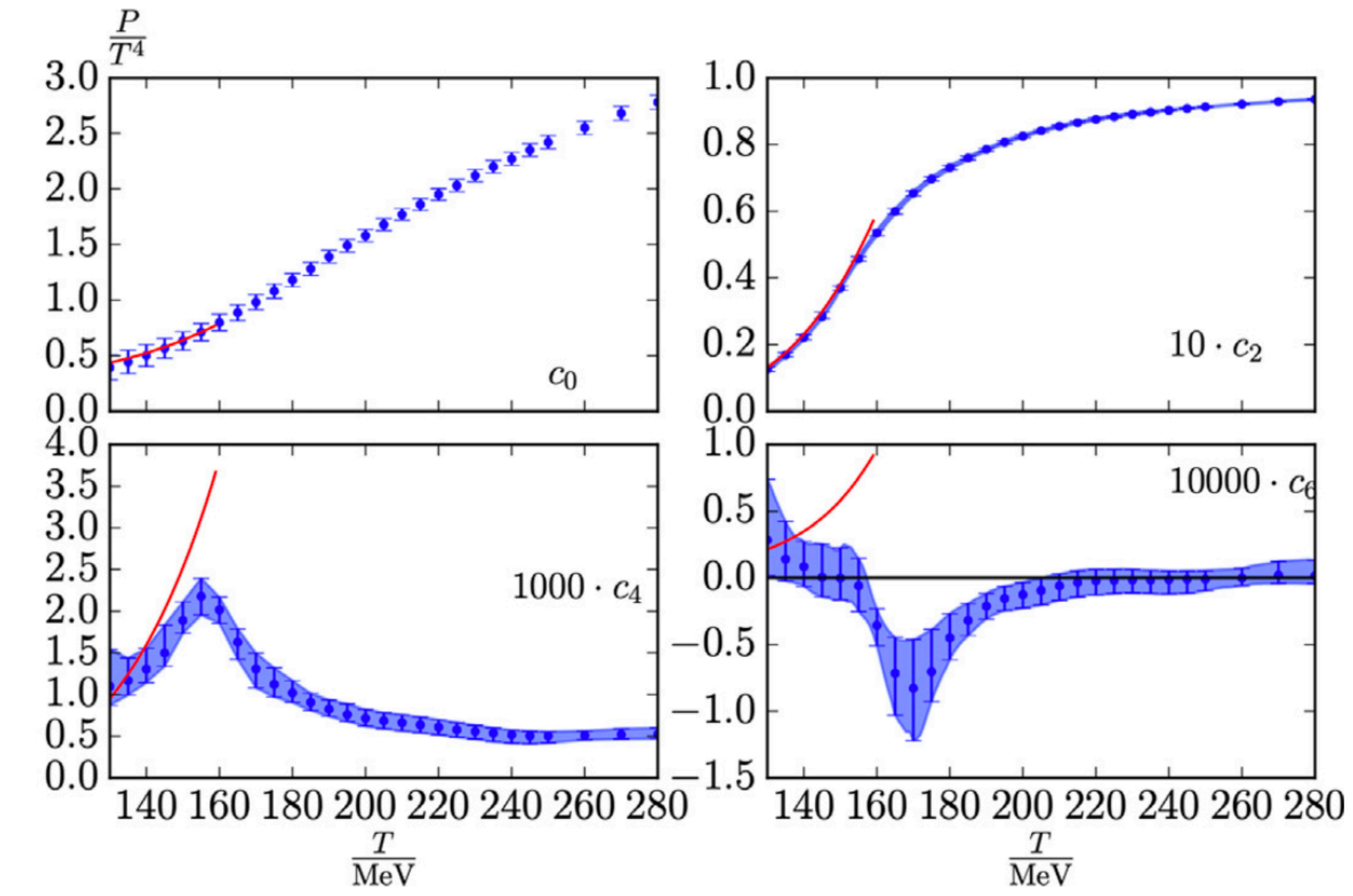


LRP 2007

S. Borsányi *et al.*, Phys. Lett. B **730** 99–104 (2014) arXiv:1309.5258

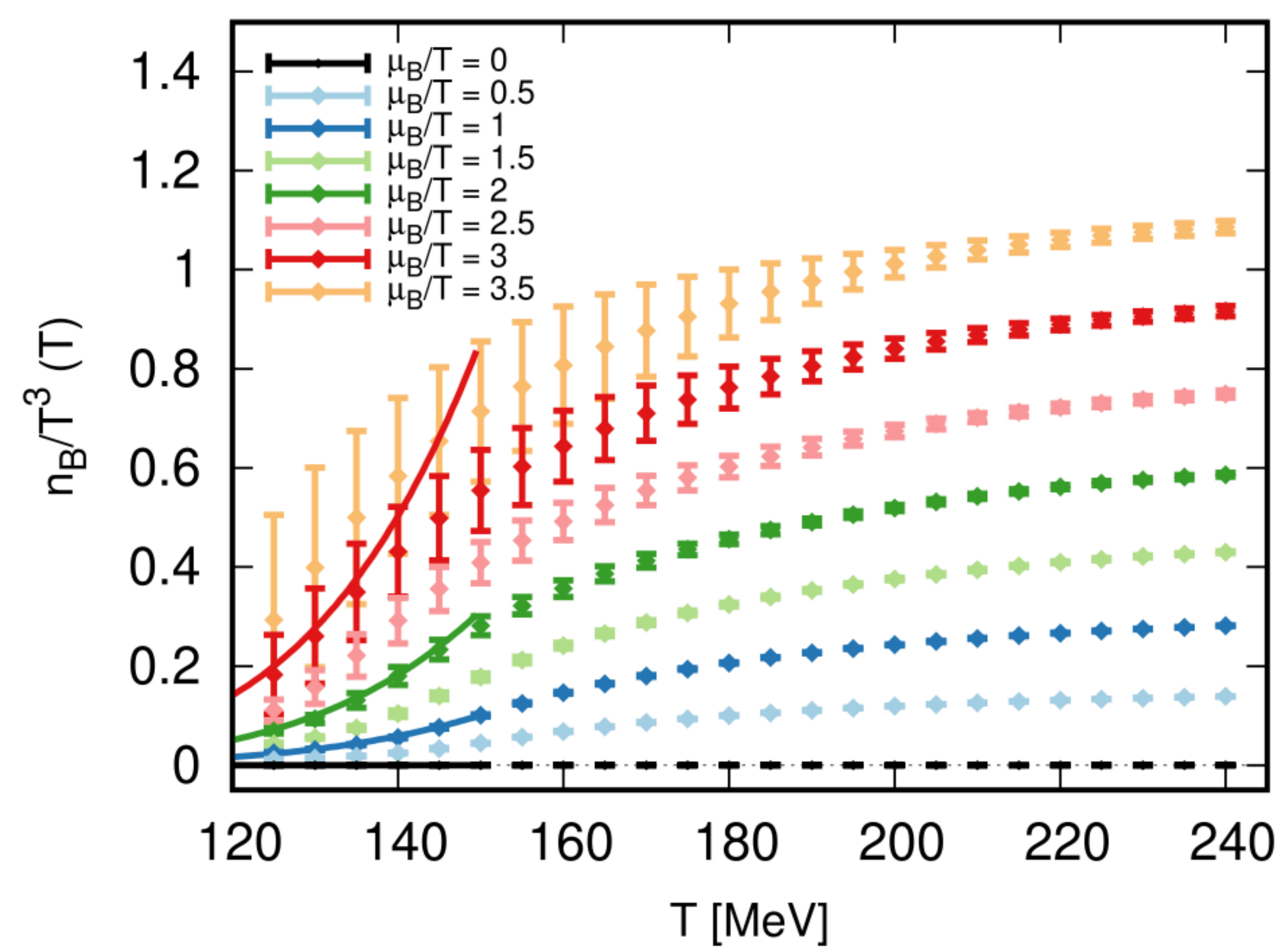
Lattice QCD EOS at finite μ_B

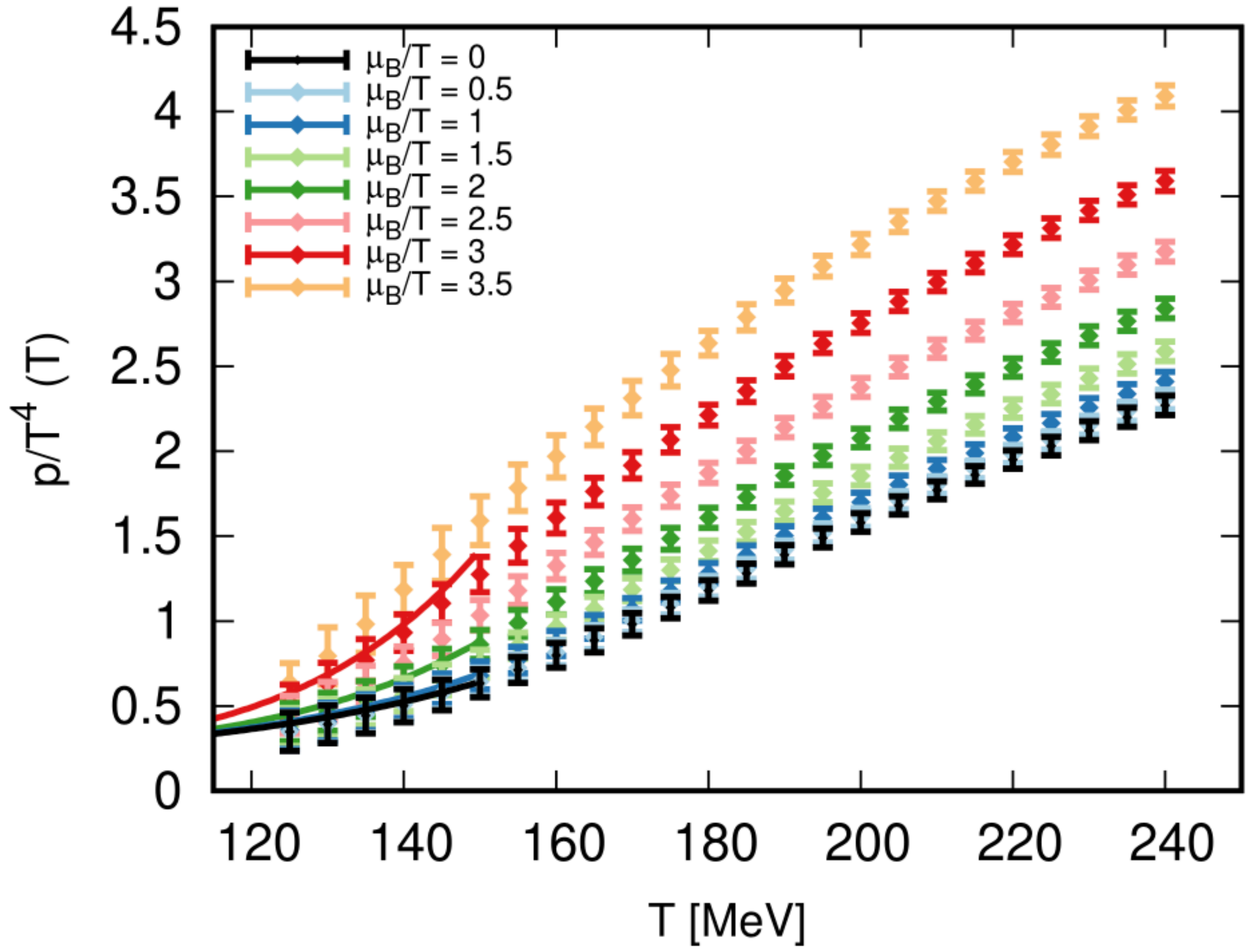


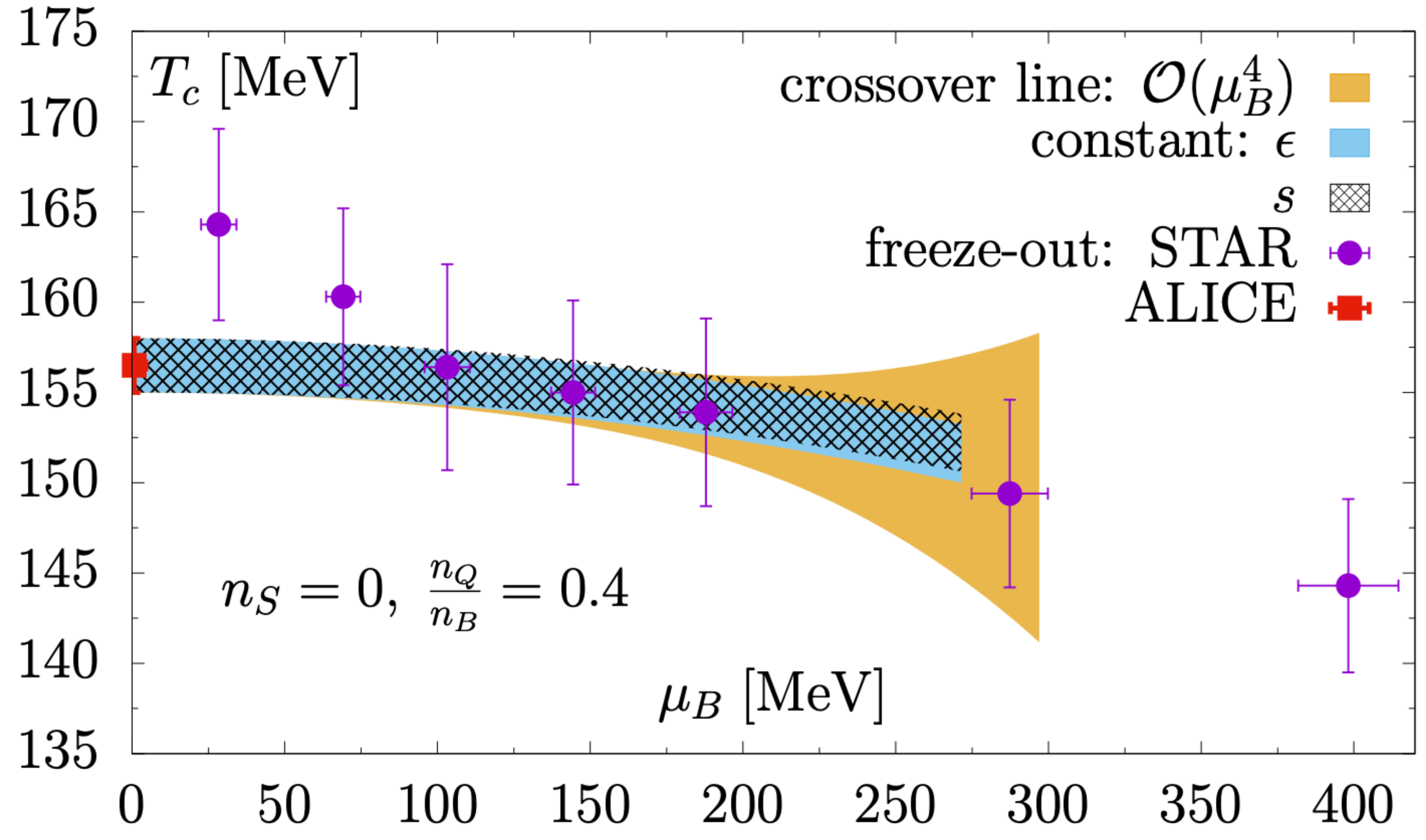


J.N. Guenther *et al.*, Nuclear Phys. A 967 (2017) 720–723 arXiv:1607.02493

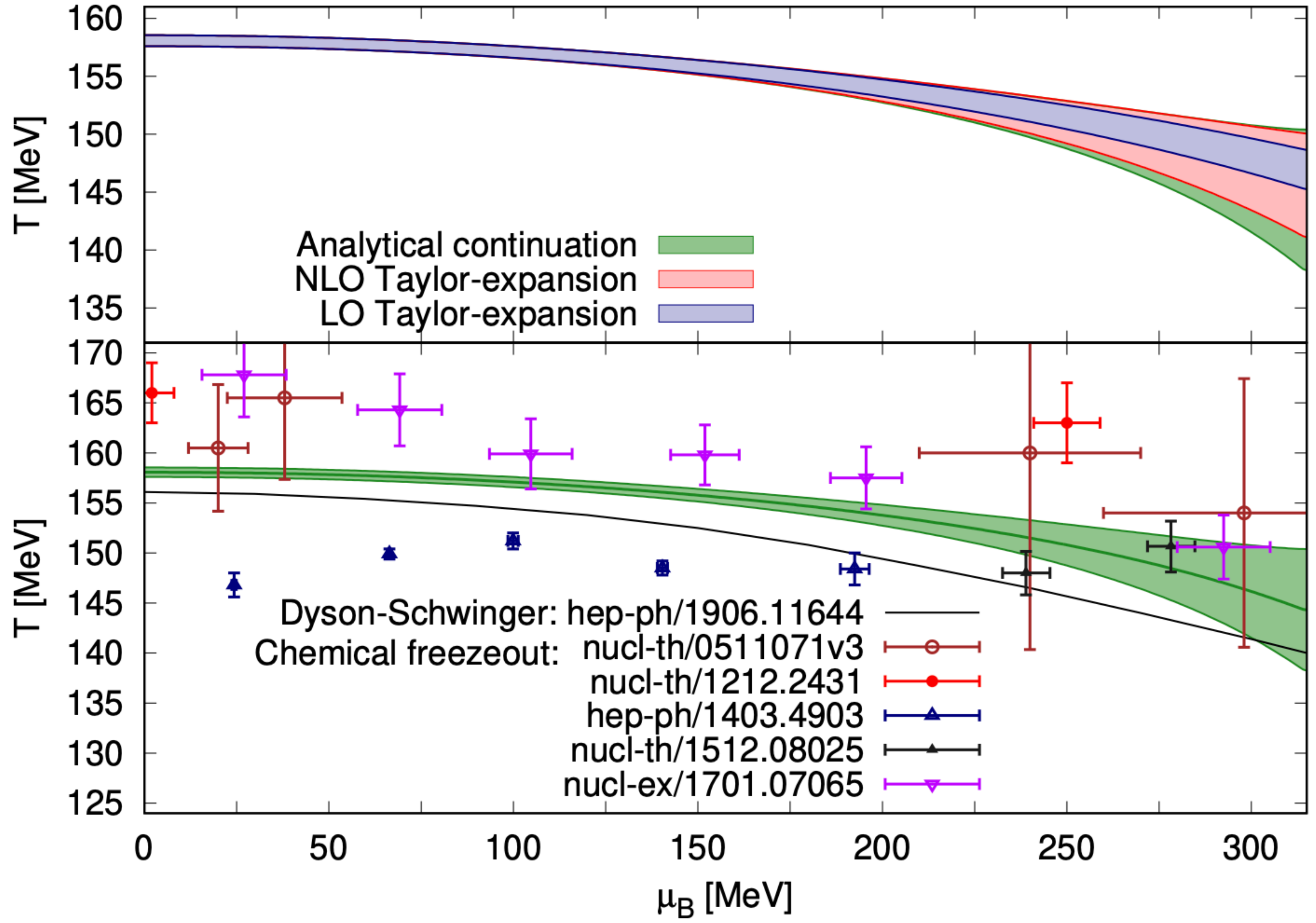
S. Borsányi *et al.*, Phys. Rev. Lett. 126 (23) (2021) 232001, arXiv:2102.06660





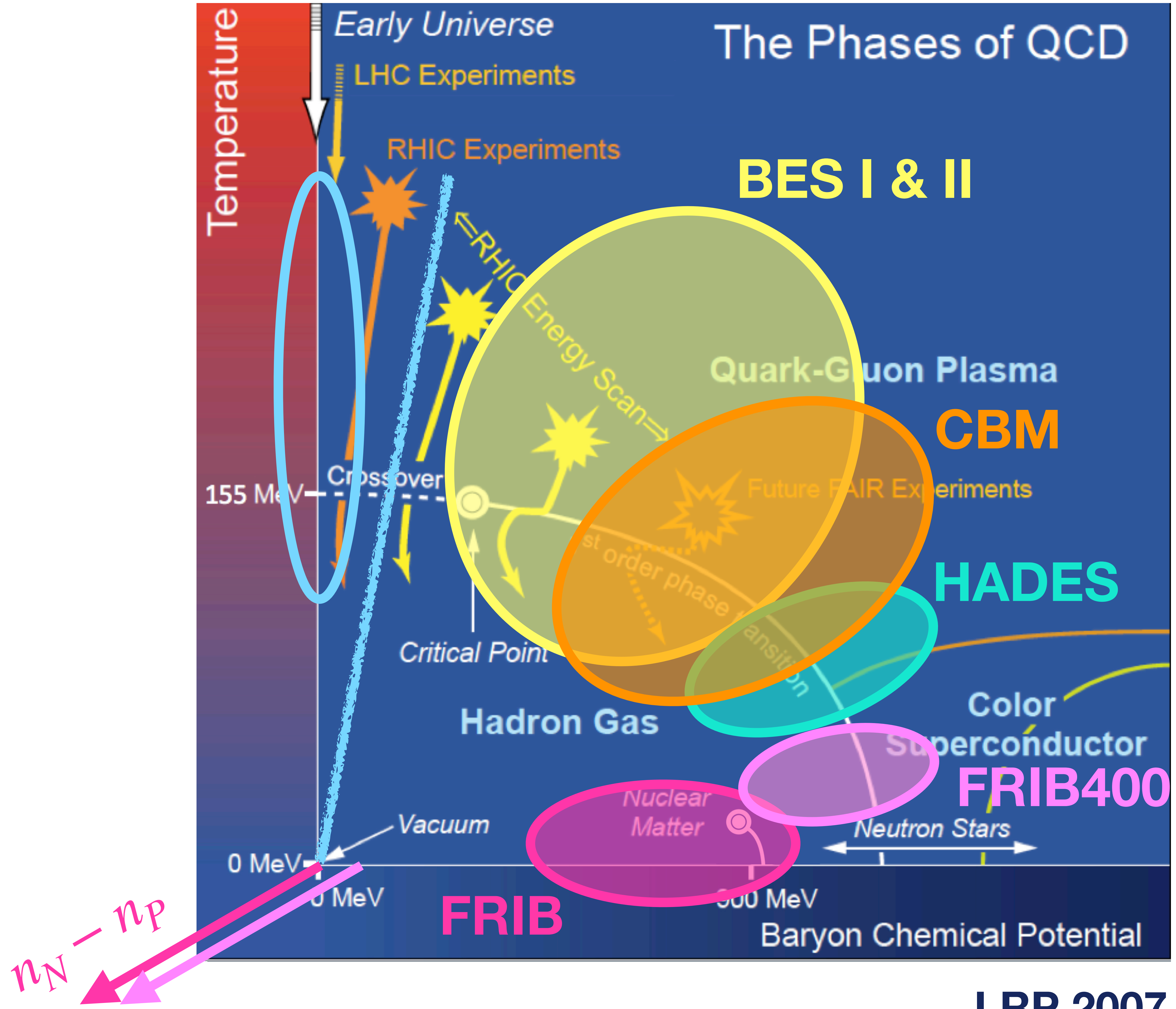


A. Bazavov *et al.* (HotQCD)
Phys. Lett. B **795**, 15 (2019), arXiv:1812.08235



S. Borsányi *et al.*, Phys. Rev. Lett. **125**, 052001 (2020), arXiv:2002.02821

The QCD phase diagram: what happens at at high μ_B?

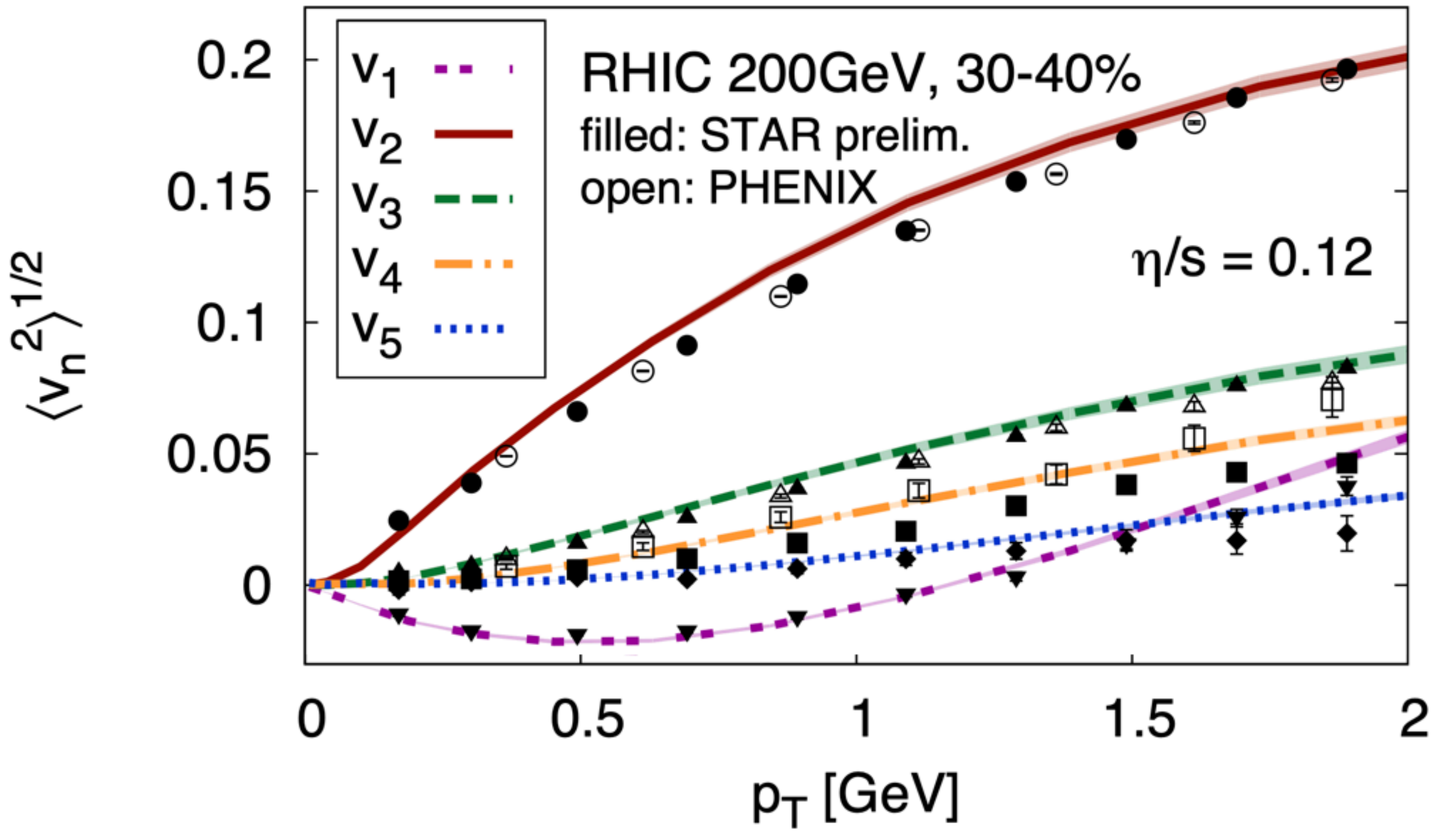


LRP 2007

Models predict a 1st order phase transition at large μ_B ~ large n_B

The EOS of dense nuclear matter in heavy-ion collisions

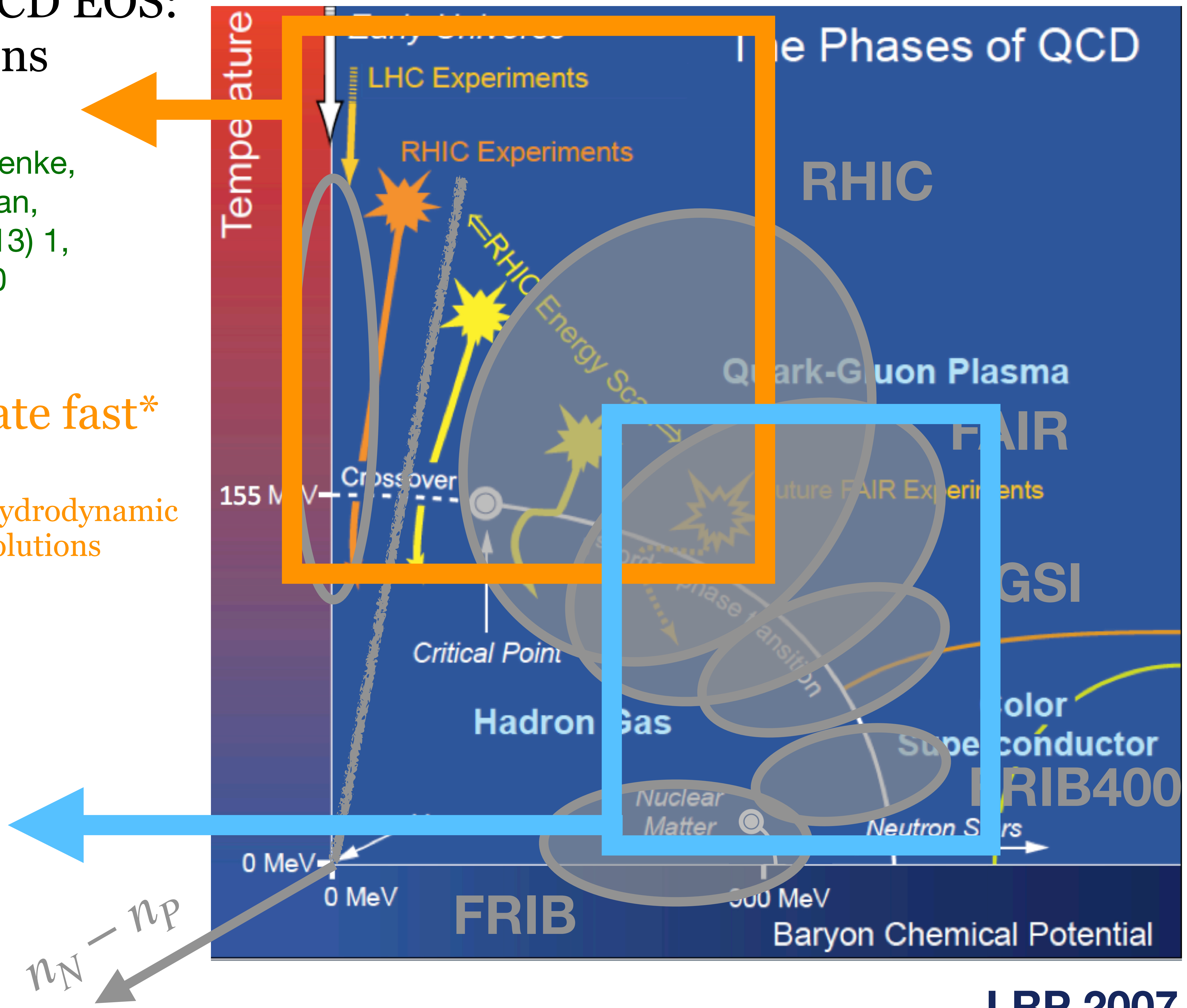
Relativistic viscous hydrodynamic simulations with LQCD EOS: amazing agreement with data from high-energy collisions



C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. **110** (2013) 1, 012302, arXiv:1209.6330

systems equilibrate fast*
= hydro applies

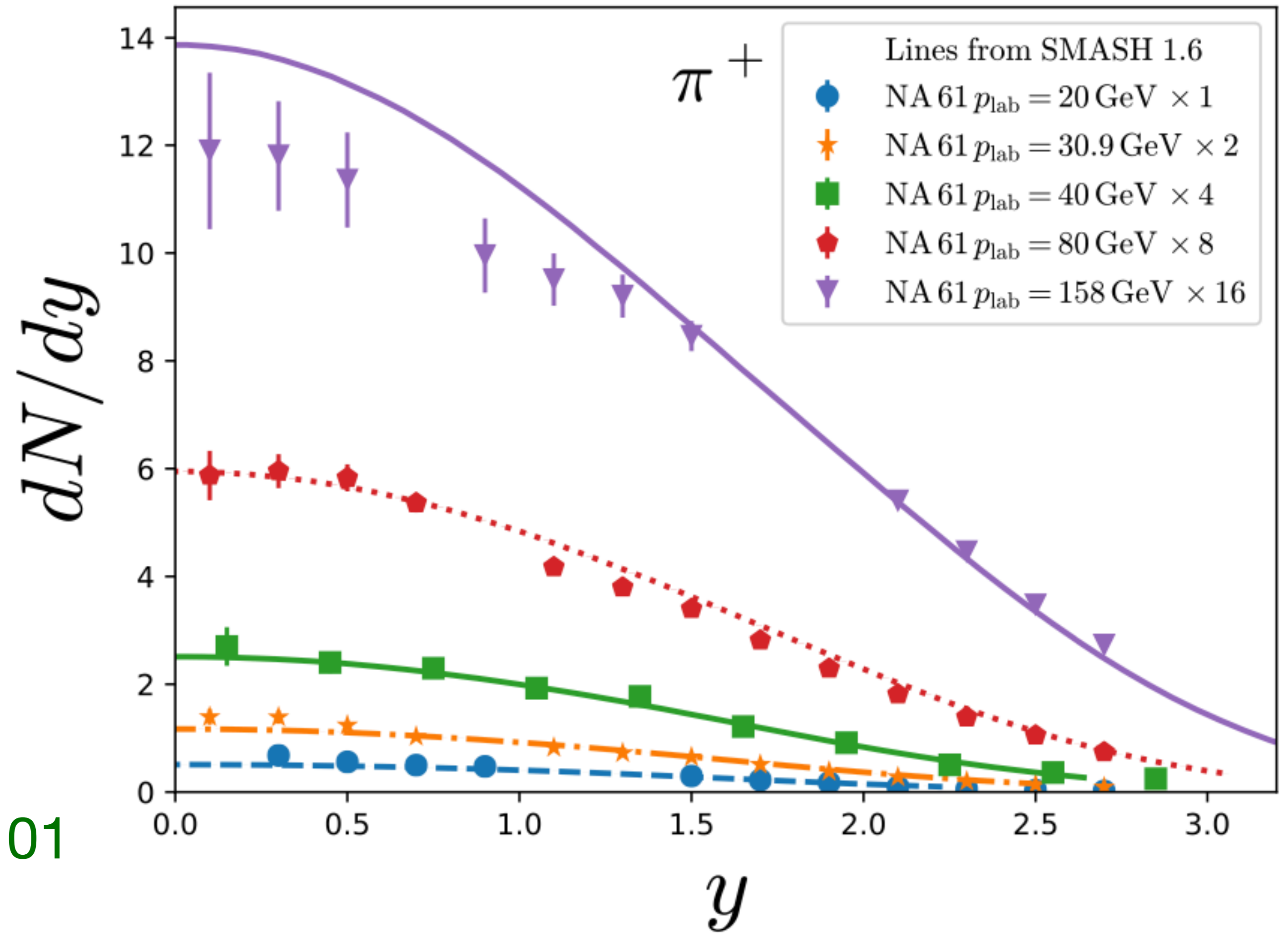
* and even if they don't, hydrodynamic attractors lead to hydro solutions



LRP 2007

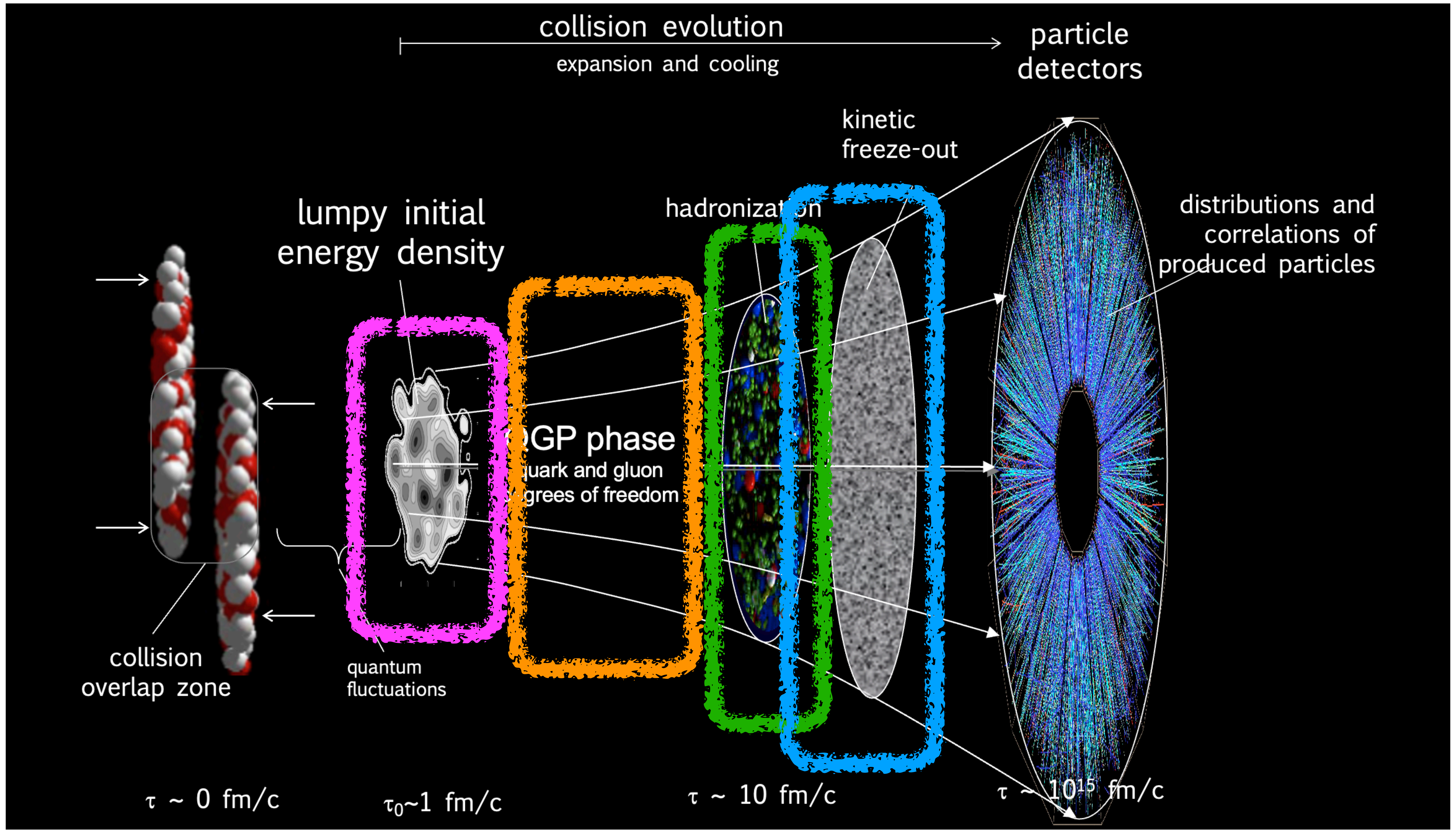
Hadronic transport simulations:

systems out of equilibrium
= microscopic approach needed



J. Mohs, S. Ryu, H. Elfner, J. Phys. G **47** (2020) 6, 065101 arXiv:1909.05586

Stages of a heavy-ion collision



P. Sorensen, Quark-gluon plasma 4, 323–374 (2010) arXiv:0905.0174

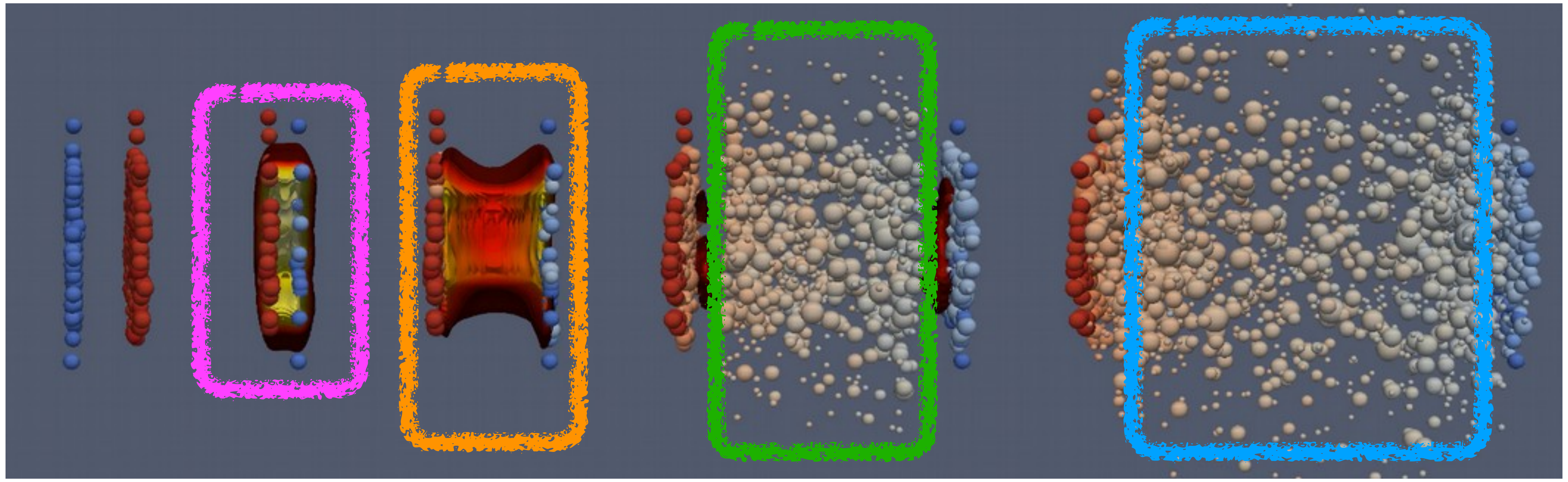
impact ~ initial state — collision geometry, collision energy, nuclear structure

expansion — hydrodynamics: driven by the EOS; transport: driven by the EOS & scatterings

hadronization

loss of information?

hadronic evolution & freeze-out

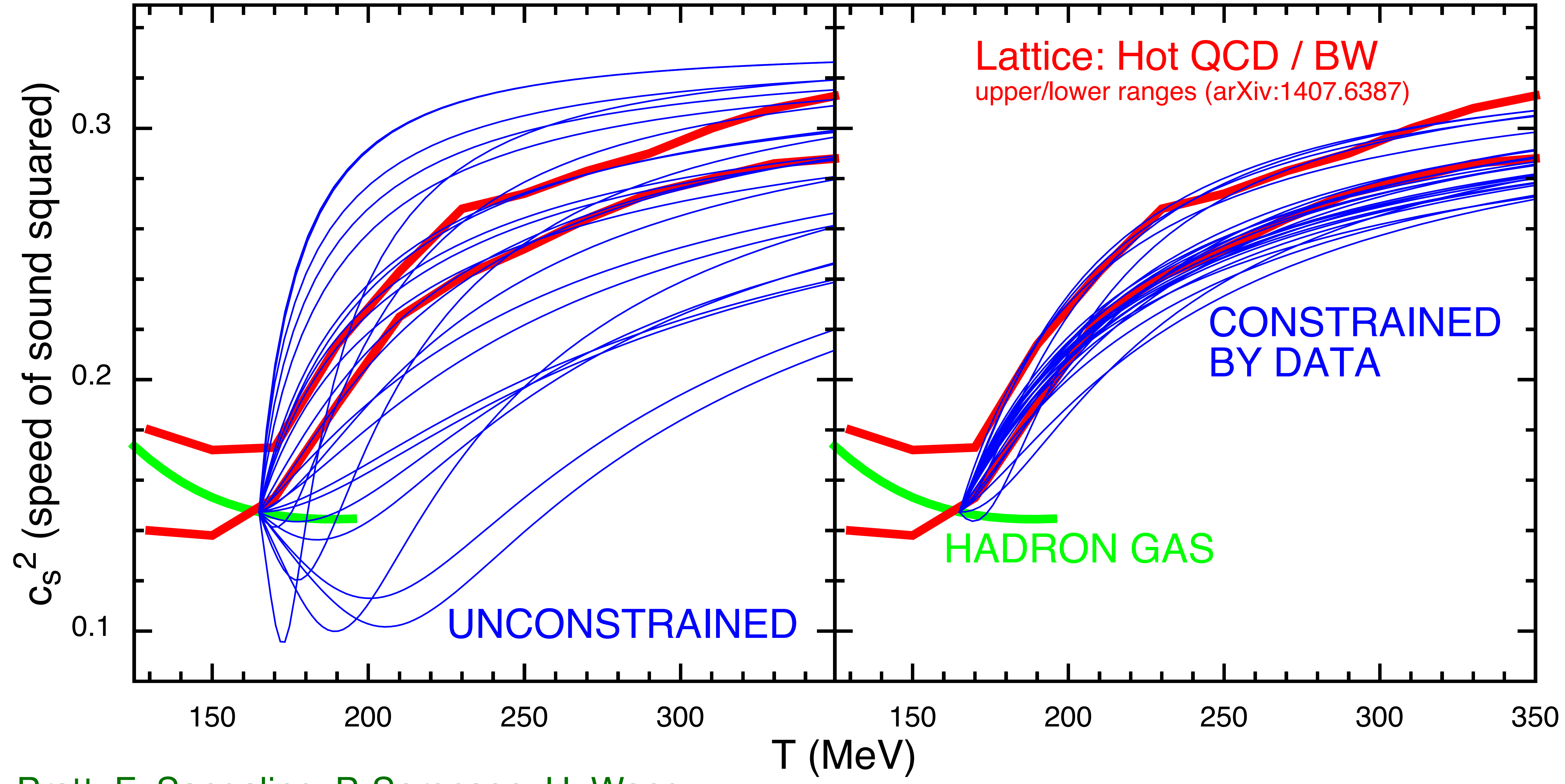


MADAI collaboration, http://madai.us

Objective of BES: the EOS and the phase diagram

Use heavy-ion collisions to study the QCD EOS = extract equilibrium bulk properties
from an extremely small ($\sim 10^{-14}$ m across)
and extremely short-lived ($\sim 10^{-22}$ s) system
using phenomenological simulations

Is it even possible??? What we learned at top RHIC energies suggests YES!

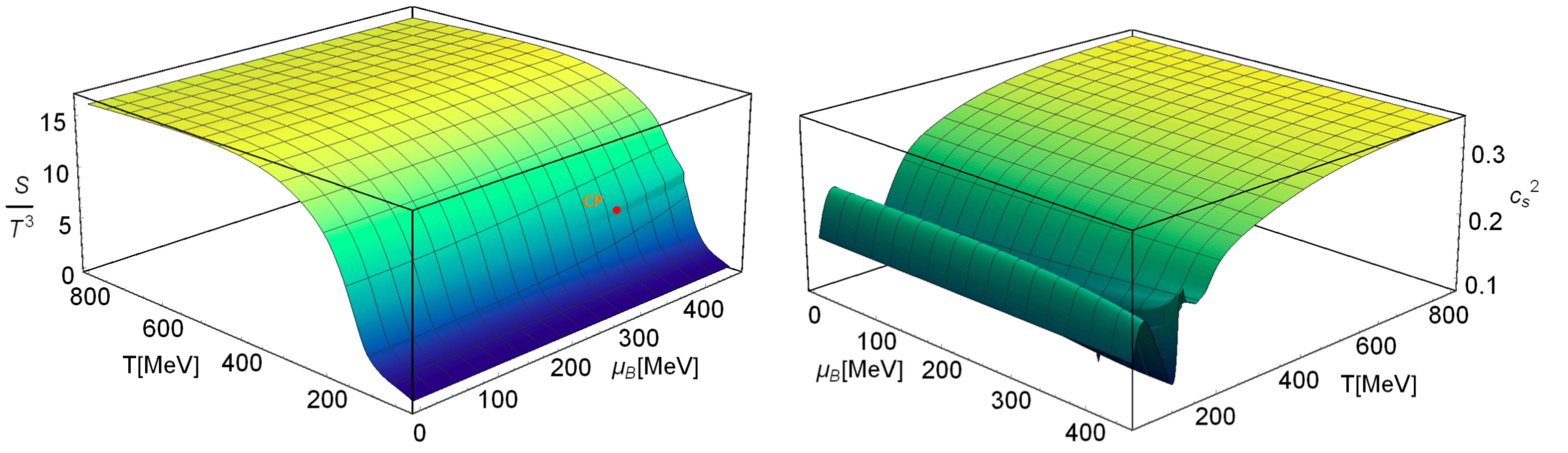


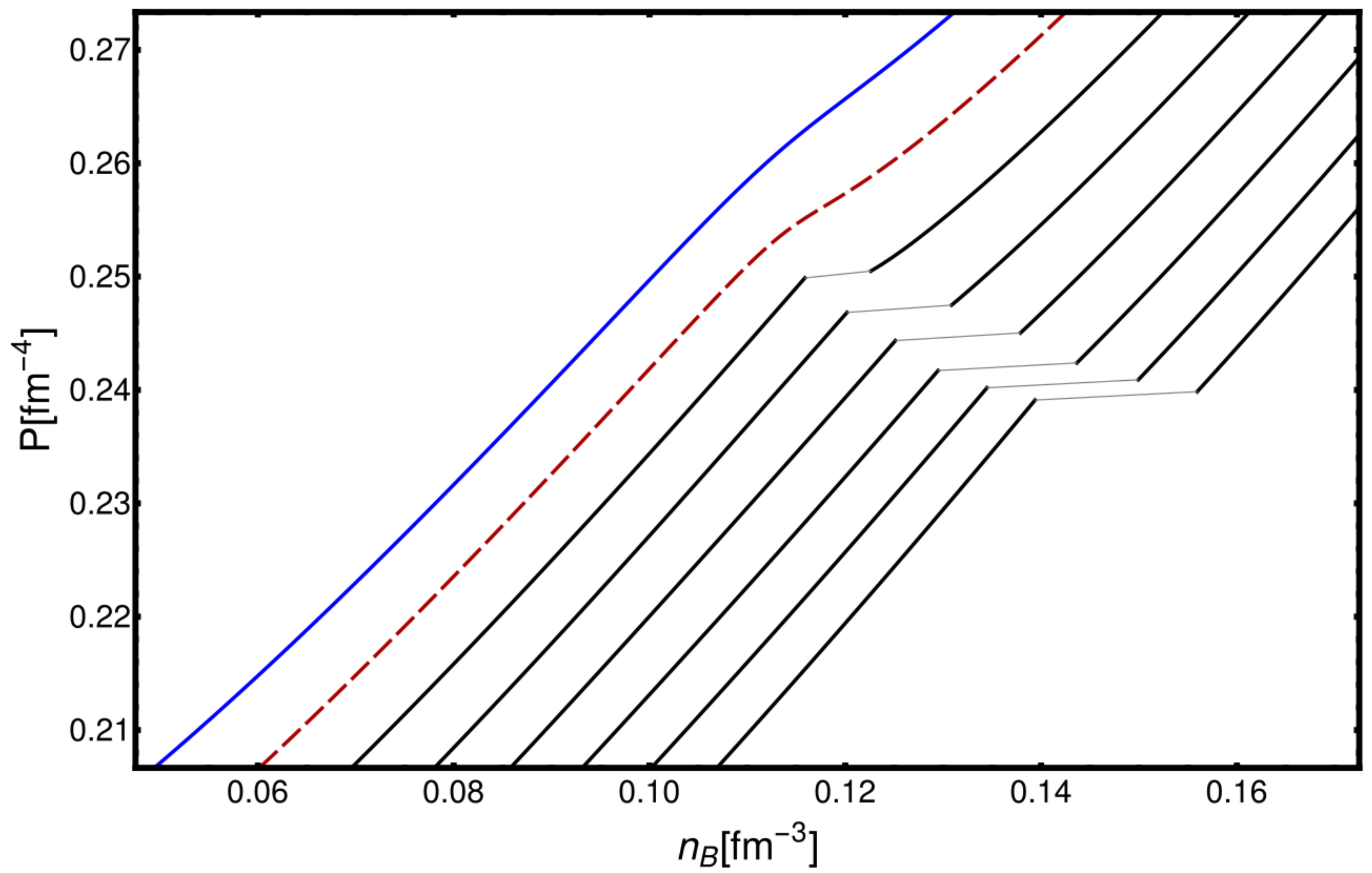
S. Pratt, E. Sangaline, P. Sorensen, H. Wang,
Phys. Rev. Lett. **114** 202301 (2015), arXiv:1501.04042

EOS constrained by Bayesian analysis of heavy-ion collisions at top RHIC energy ($\mu_B \approx 0$) agrees with LQCD

No/Scarce theory predictions at finite μ_B
=
Unique occasion to guide theory and understanding of QCD by extracting the EOS from new experimental data

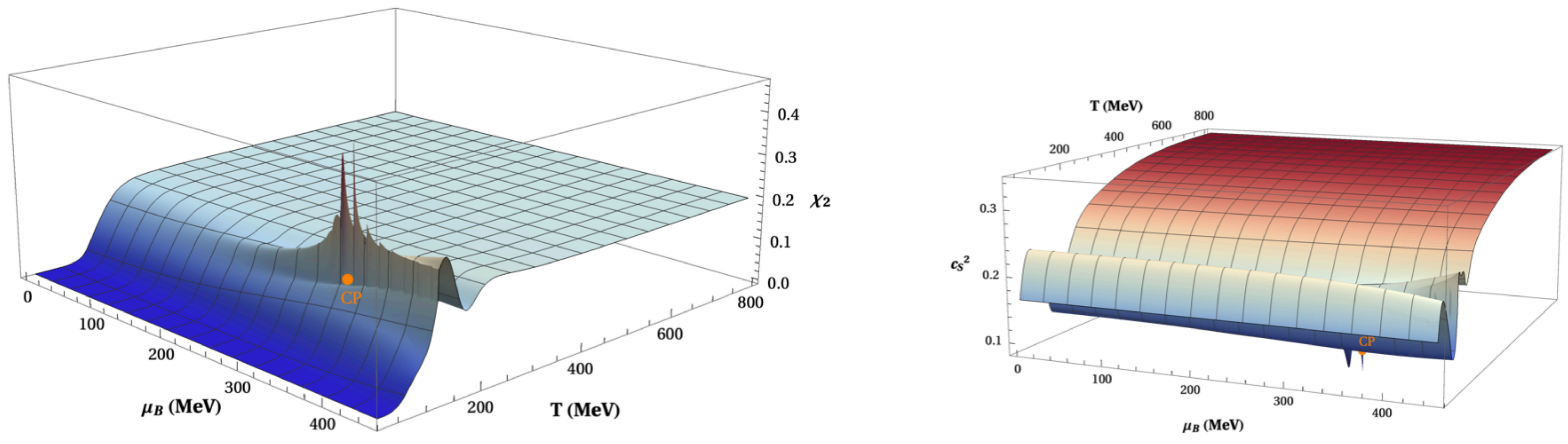
Input to hydrodynamics: EOS with 3D-Ising model critical point





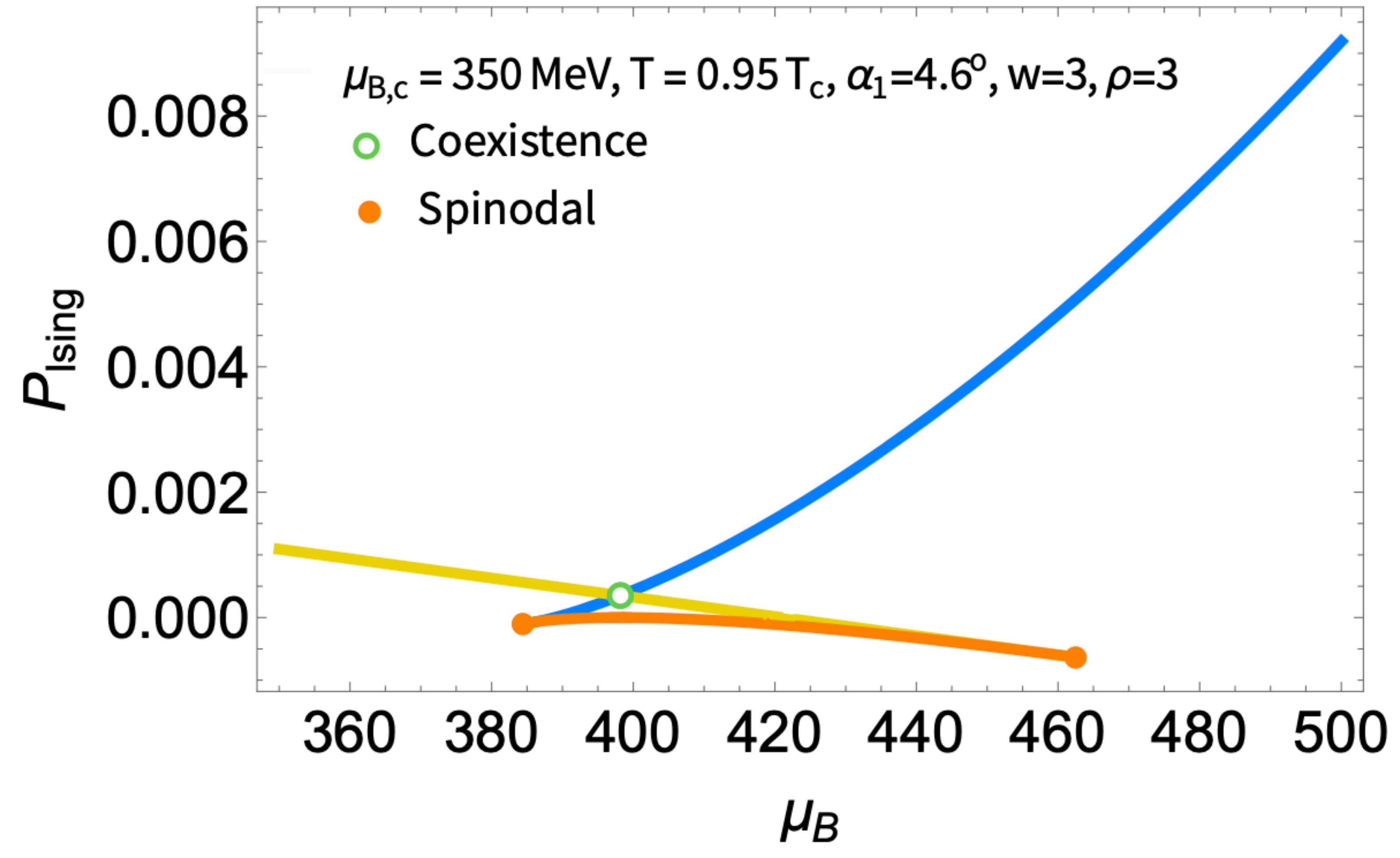
P. Parotto *et al.*, Phys. Rev. C 101, 034901 (2020), arXiv:1805.05249

with strangeness-neutrality:



J.M. Karthein *et al.*, Eur. Phys. J. Plus **136** 6, 621 (2021) arXiv:2103.08146

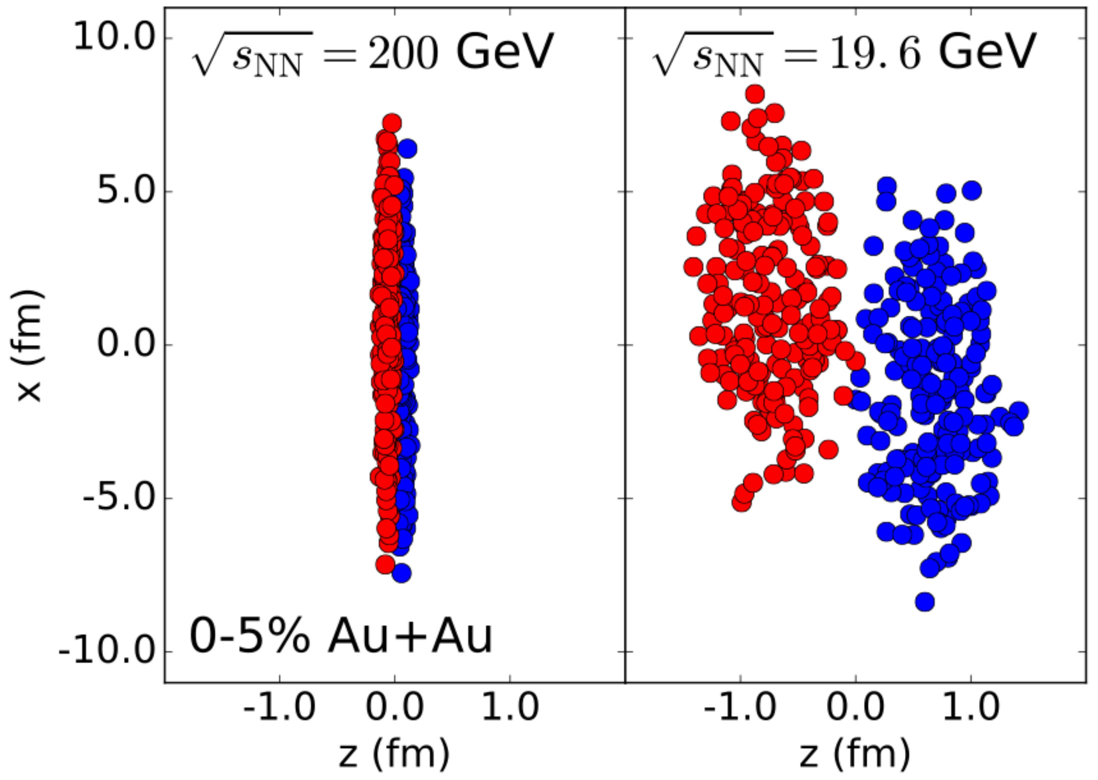
with spinodal regions:

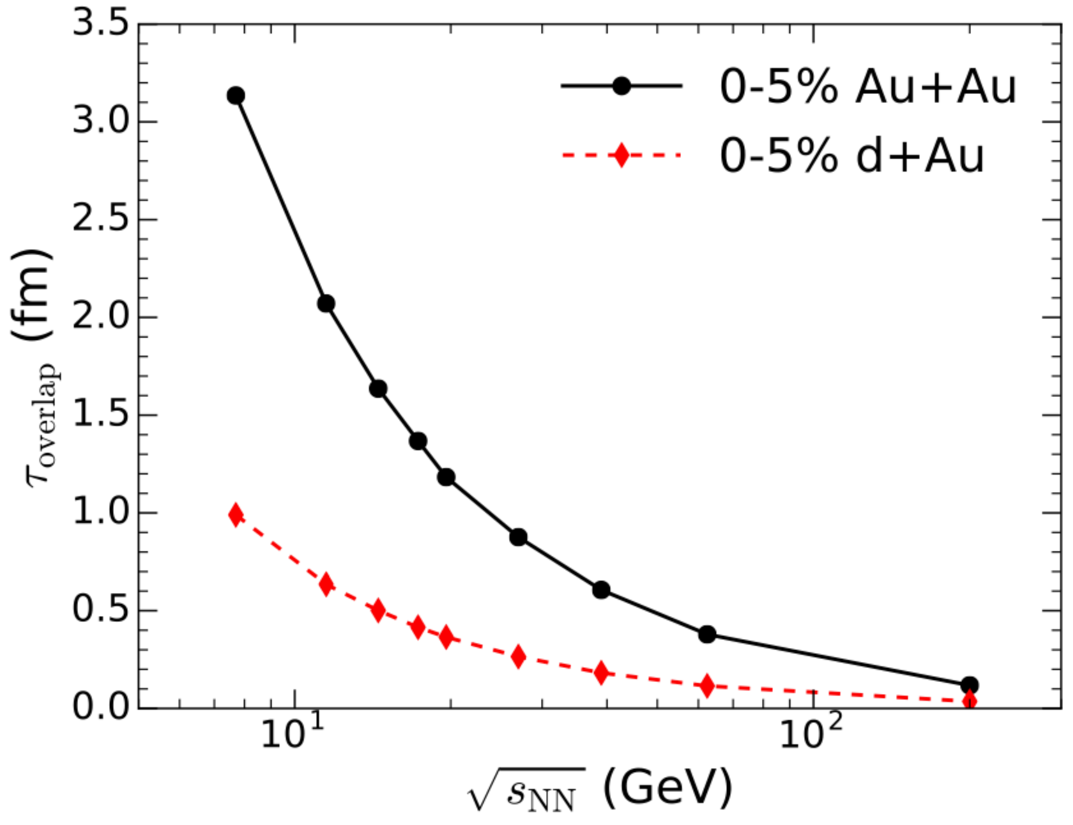


J.M. Karthein, V. Koch, C. Ratti arXiv:2409.13961

EOS is only one of many aspects of hydrodynamics

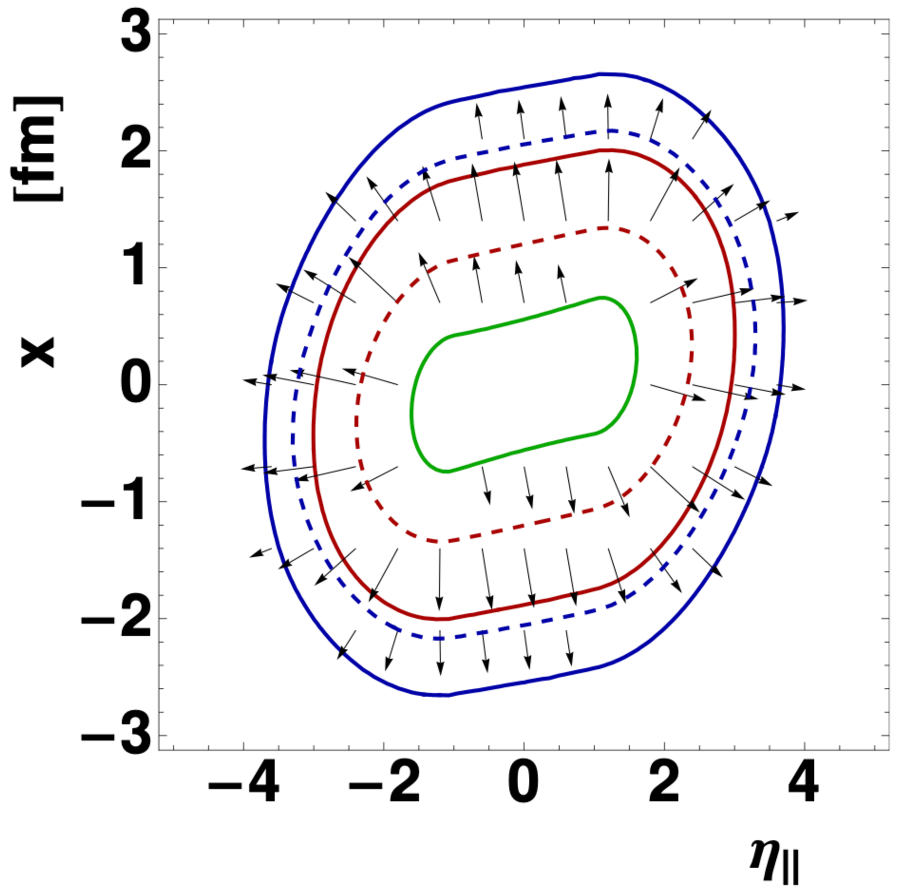
low collision energy = prolonged initial stage:

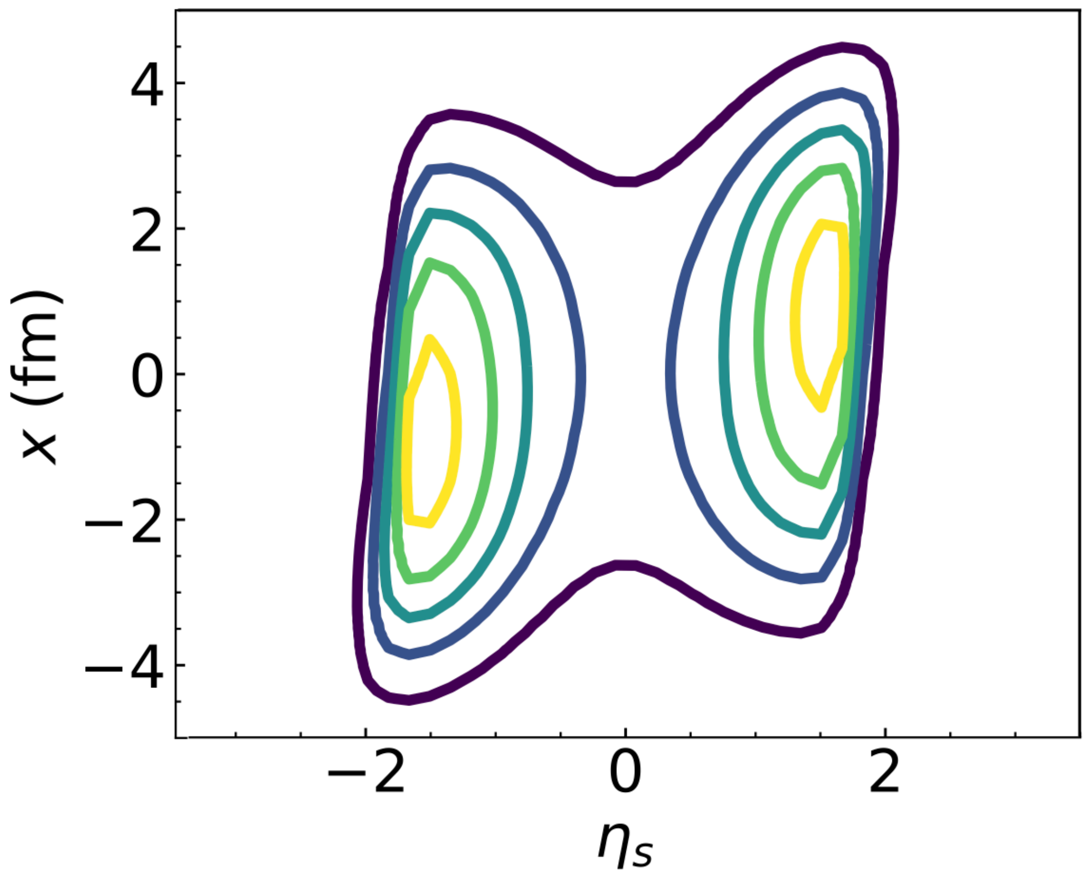




C. Shen, B. Schenke, Phys. Rev. C. **97** (2), 024907 (2018) arXiv:1710.00881

parametric initial distributions for energy and baryon density:

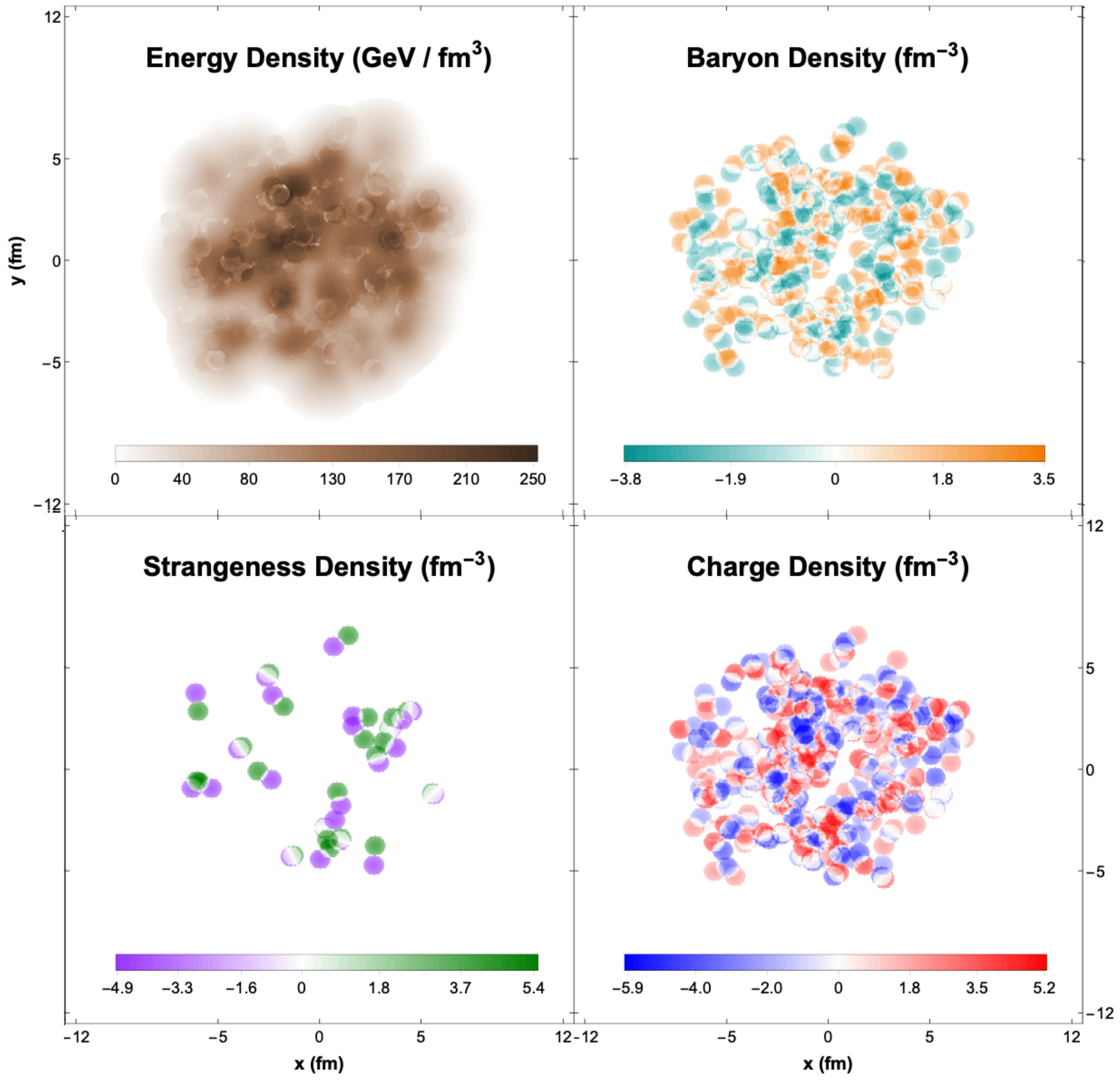




P. Bozek, I. Wyskiel, Phys. Rev. C. 81, 054902 (2010) arXiv:1002.4999

L. Du, C. Shen, S. Jeon, C. Gale, Phys. Rev. C. 108(4), L041901 (2023) arXiv:2211.16408

multiple conserved charges initialized with ICCING (Initial Conserved Charges in Nuclear Geometry)



P. Carzon *et al.*, Phys. Rev. C. 105(3), 034908 (2022) arXiv:1911.12454
P. Carzon *et al.*, Phys. Rev. C. 108 (6), 064905 (2023) arXiv:2301.04572

EOS is only one of many aspects of hydrodynamics

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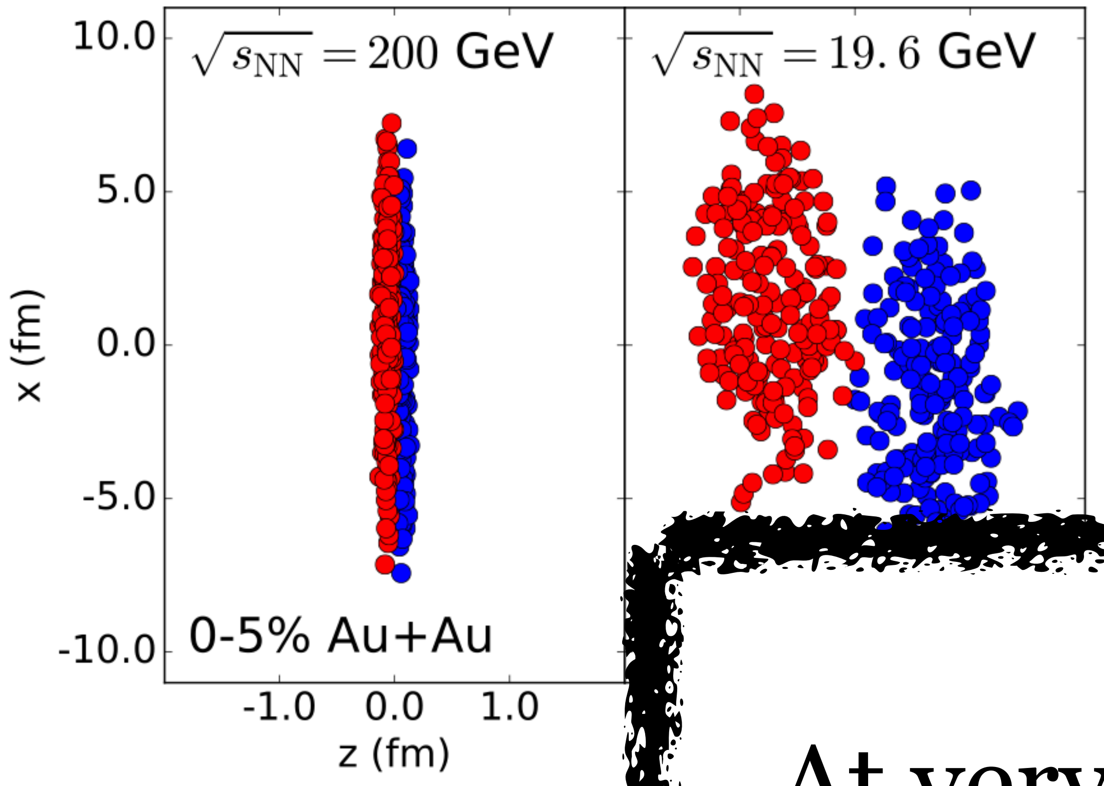

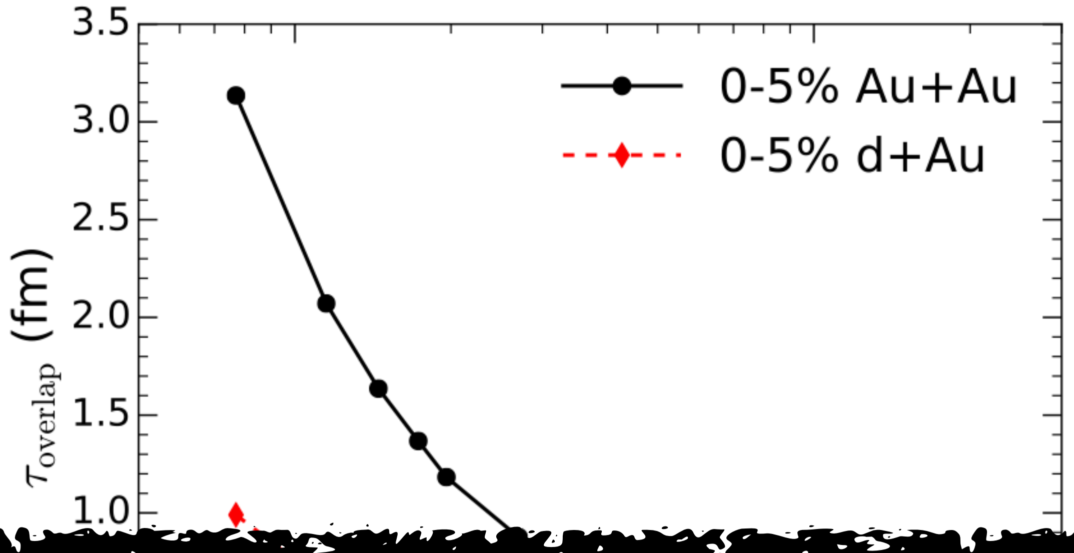

C. Shen, B. Schen

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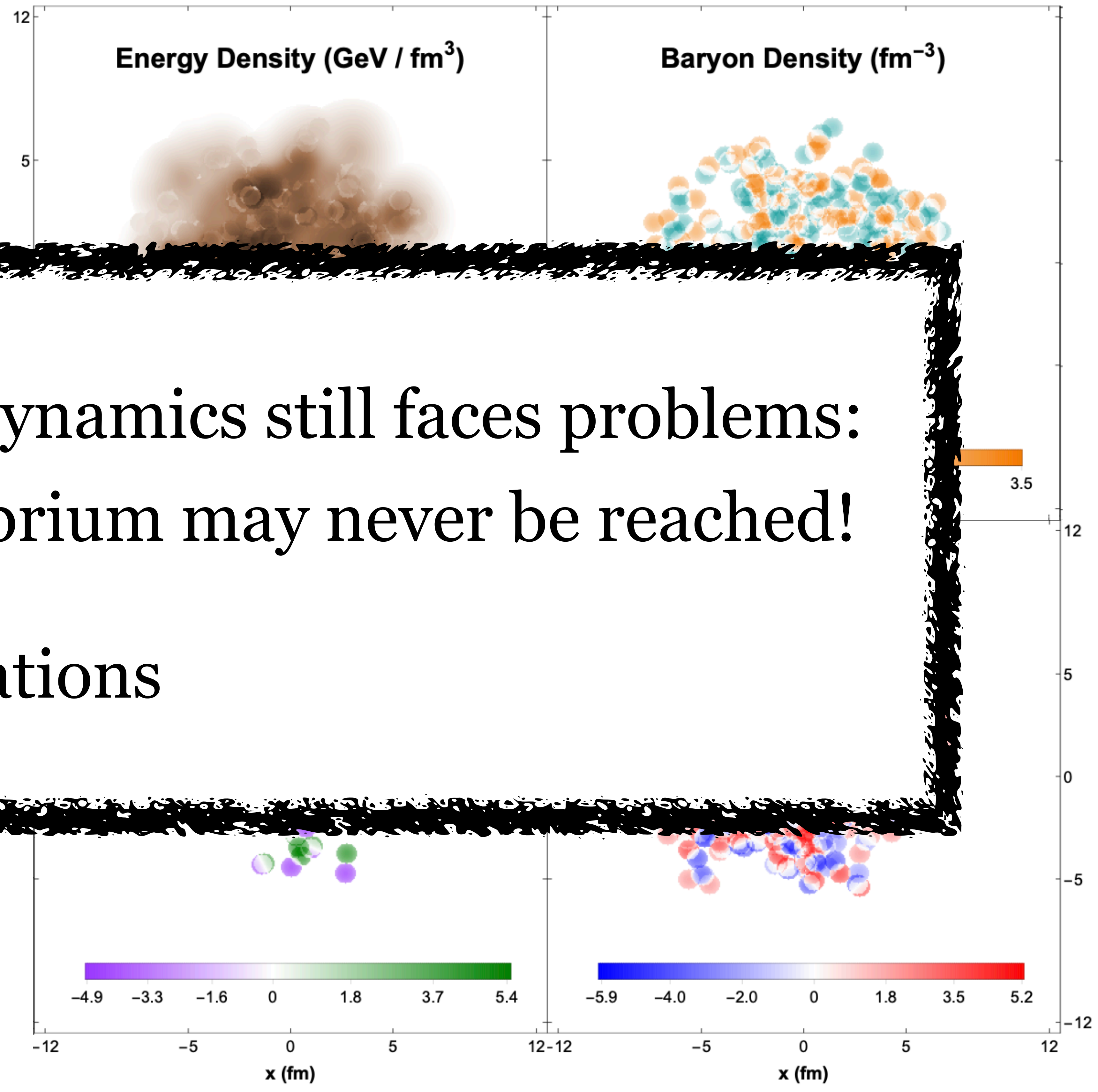
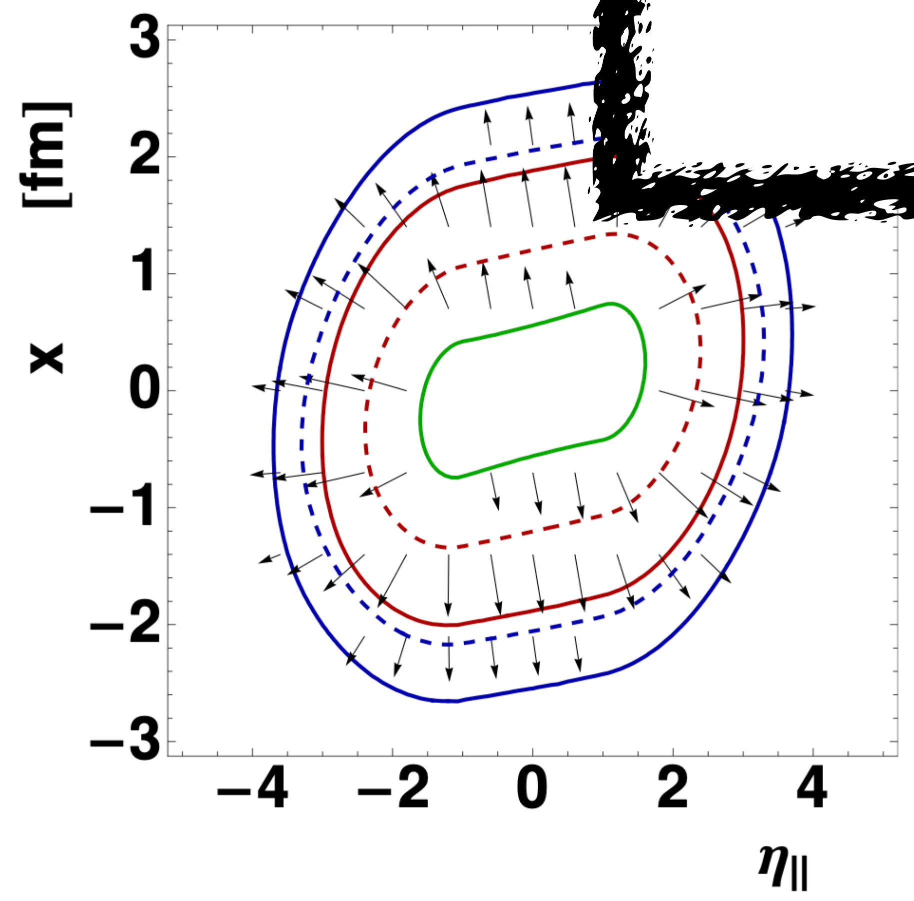


P. Carzon *et al.*, Phys. Rev. C. 105(3), 034908 (2022) arXiv:1911.12454
P. Carzon *et al.*, Phys. Rev. C. 108 (6), 064905 (2023) arXiv:2301.04572

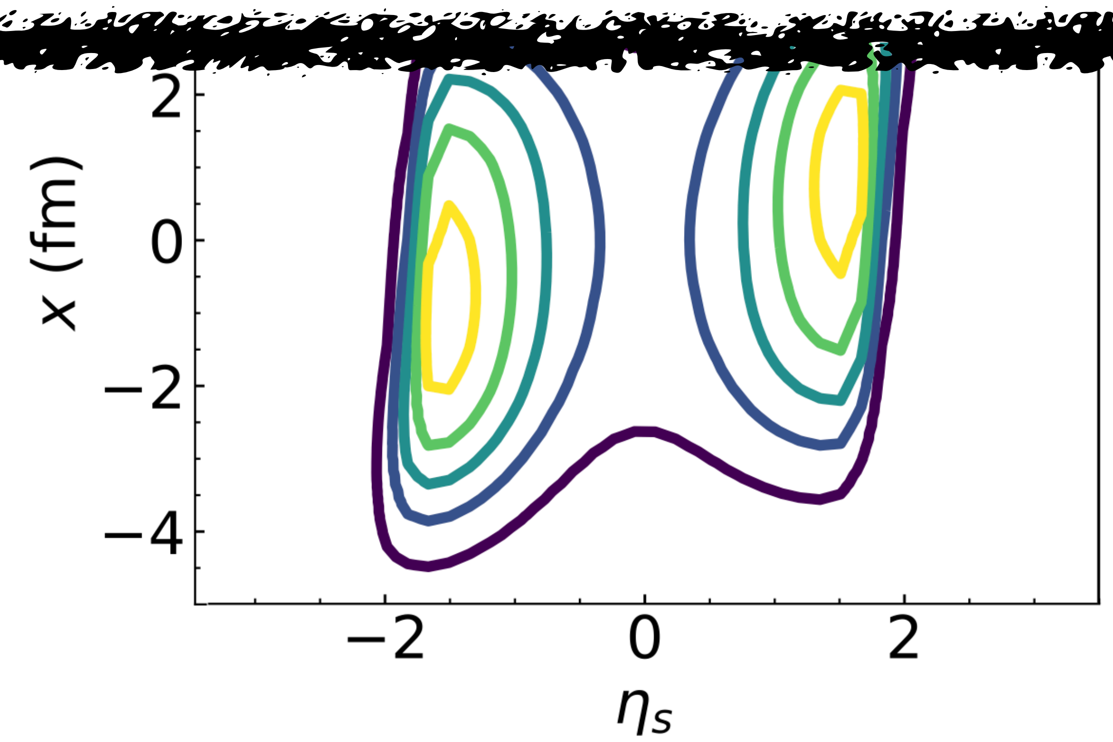
At very low energies ($\sqrt{s_{NN}} \leq 7$ GeV) hydrodynamics still faces problems: approach to equilibrium is so long that equilibrium may never be reached!

Enter transport simulations

parametric
energy and



P. Bozek, I. Wyskiel, Phys. Rev. C. 81, 054902 (2010) arXiv:1002.4999

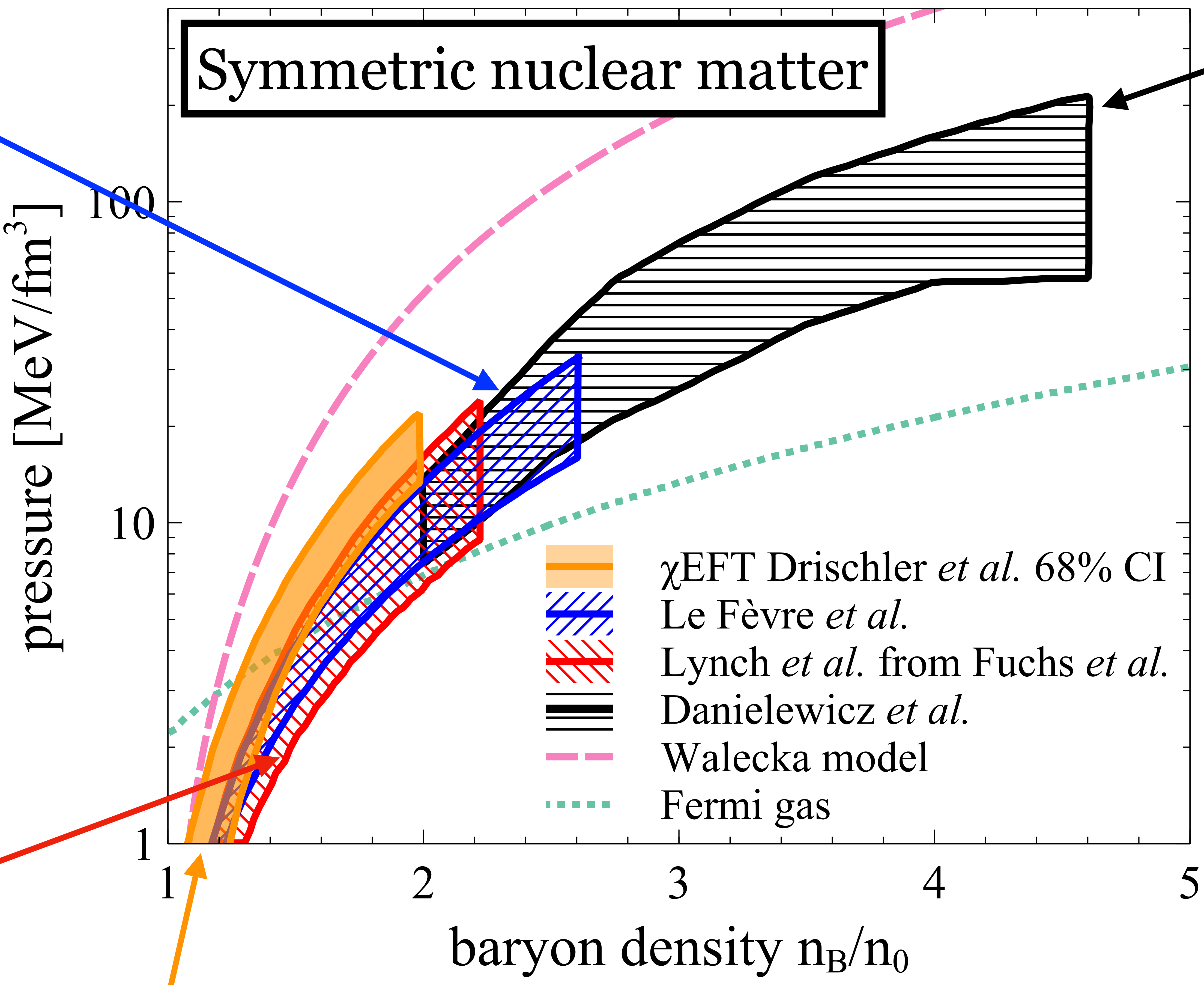


L. Du, C. Shen, S. Jeon, C. Gale, Phys. Rev. C. 108(4), L041901 (2023) arXiv:2211.16408

Constraints on the EOS come from comparisons to transport models

197Au+197Au @ 0.4—1.5 GeV/u
($\sqrt{s_{NN}}$ = 2.07 — 2.52 GeV)
observables: proton flow (FOPI)
model used: **isospin QMD (IQMD)** w/ nucleons, Δ, N*(1440), deuterons, tritons;
EOS parametrized by K_0;
momentum dependence
A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, C. Hartnack, Nucl. Phys. A 945, 112 (2016), arXiv:1501.05246

197Au+197Au & 12C+12C @ < 1.5 GeV/u
($\sqrt{s_{NN}}$ < 2.5 GeV)
observables: subthreshold kaon production (KaoS)
model used: **QMD** w/ nucleons, Δ, N*(1440), pions, kaons;
EOS parametrized by K_0;
kaon potentials, momentum dependence
C. Fuchs *et al.*, Prog. Part. Nucl. Phys. **53**, 113–124 (2004) arXiv:nucl-th/0312052



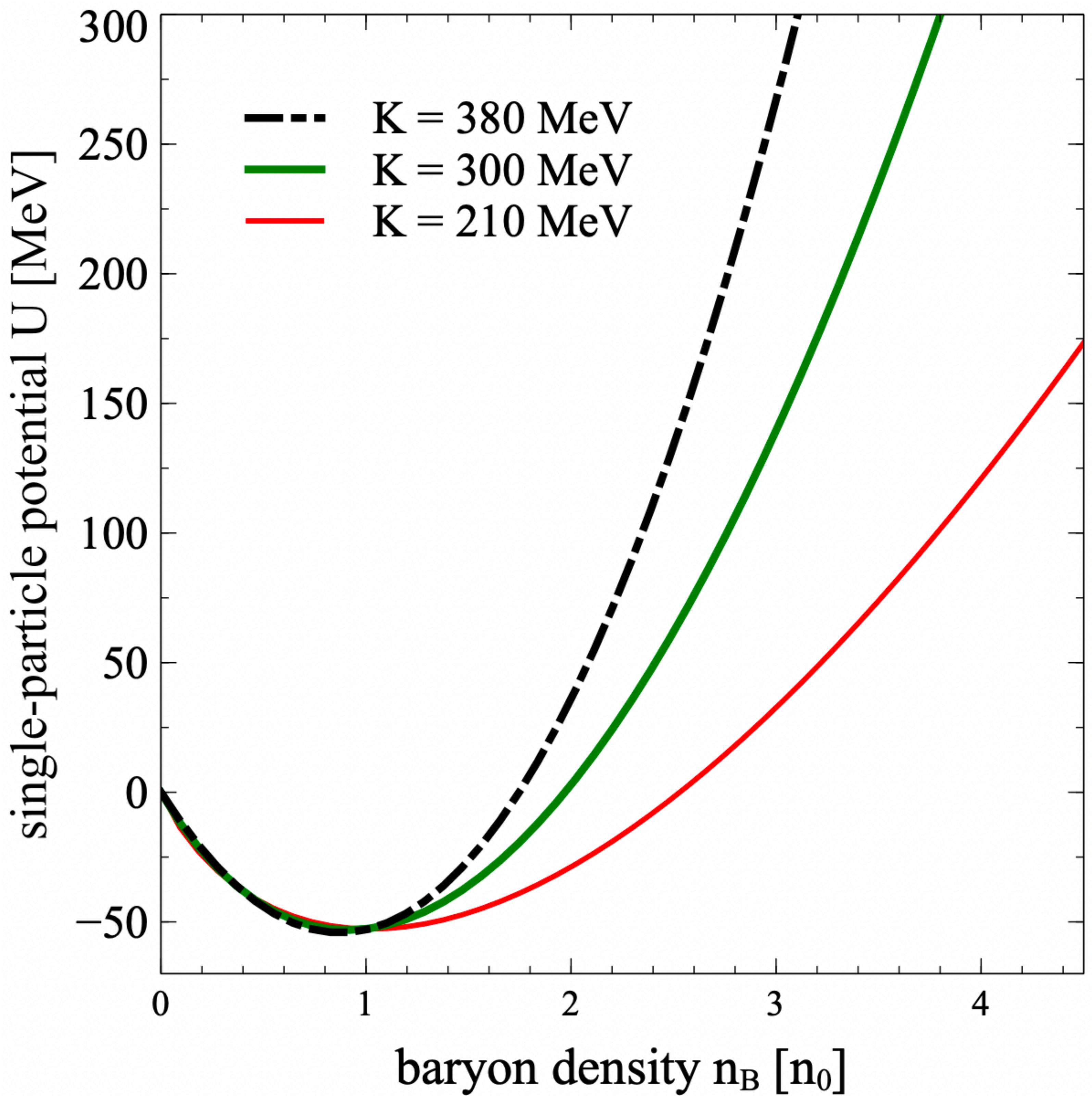
A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024)
arXiv:2301.13253

χEFT
C. Drischler *et al.*, Phys. Rev. C **102** 5, 054315 (2020)
arXiv:2004.07805

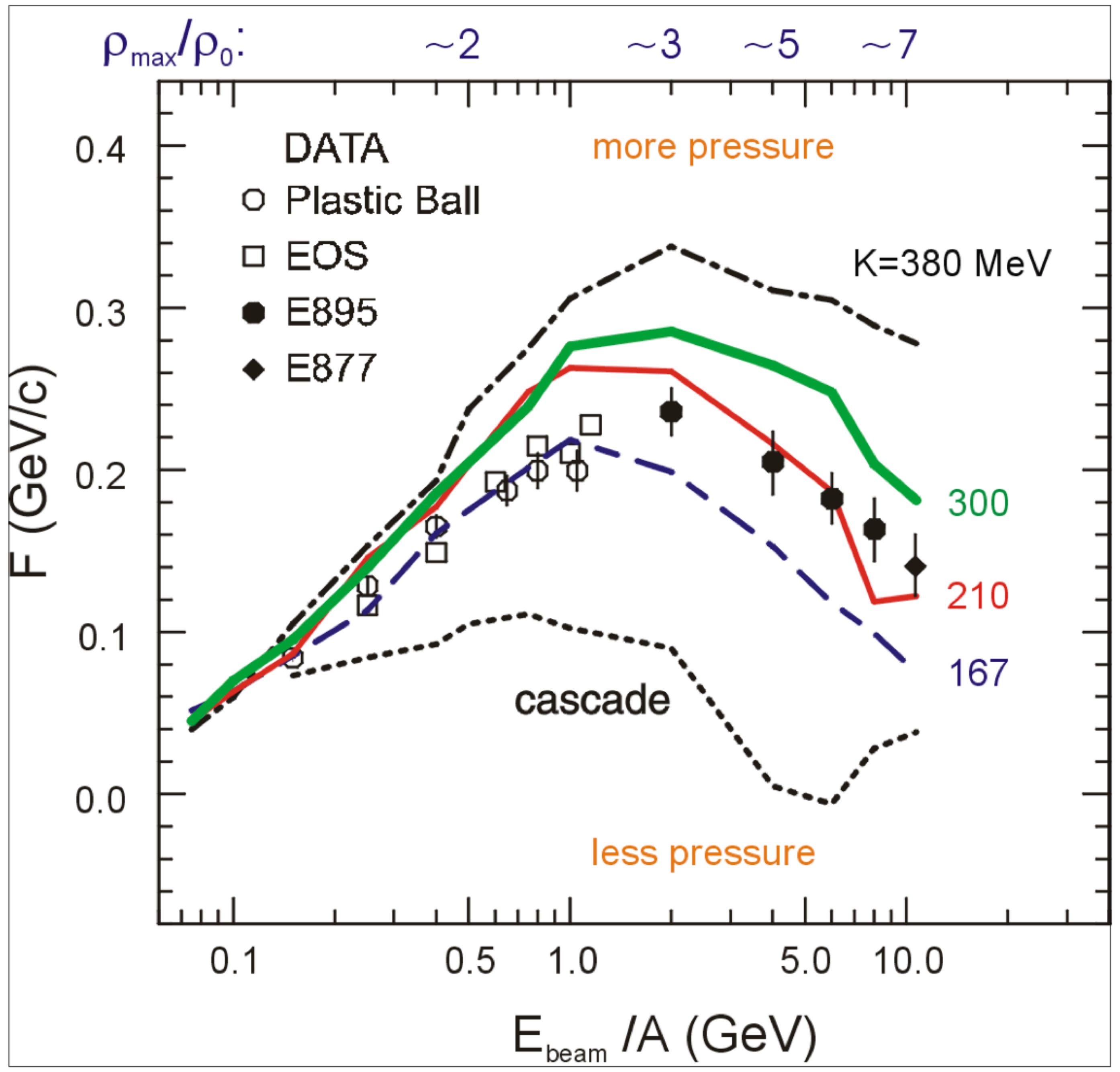
197Au+197Au @ 0.15—10 GeV/u
($\sqrt{s_{NN}}$ = 1.95 — 4.72 GeV)
observables: proton flow
(Plastic Ball, EOS, E877, E895)
model used: **pBUU** w/ nucleons, Δ, N*(1440), pions;
EOS parametrized by K_0;
momentum dependence
P. Danielewicz, R. Lacey, W. G. Lynch, Science **298**,1592–1596 (2002)

Standard way of modeling the EOS: Skyrme potential

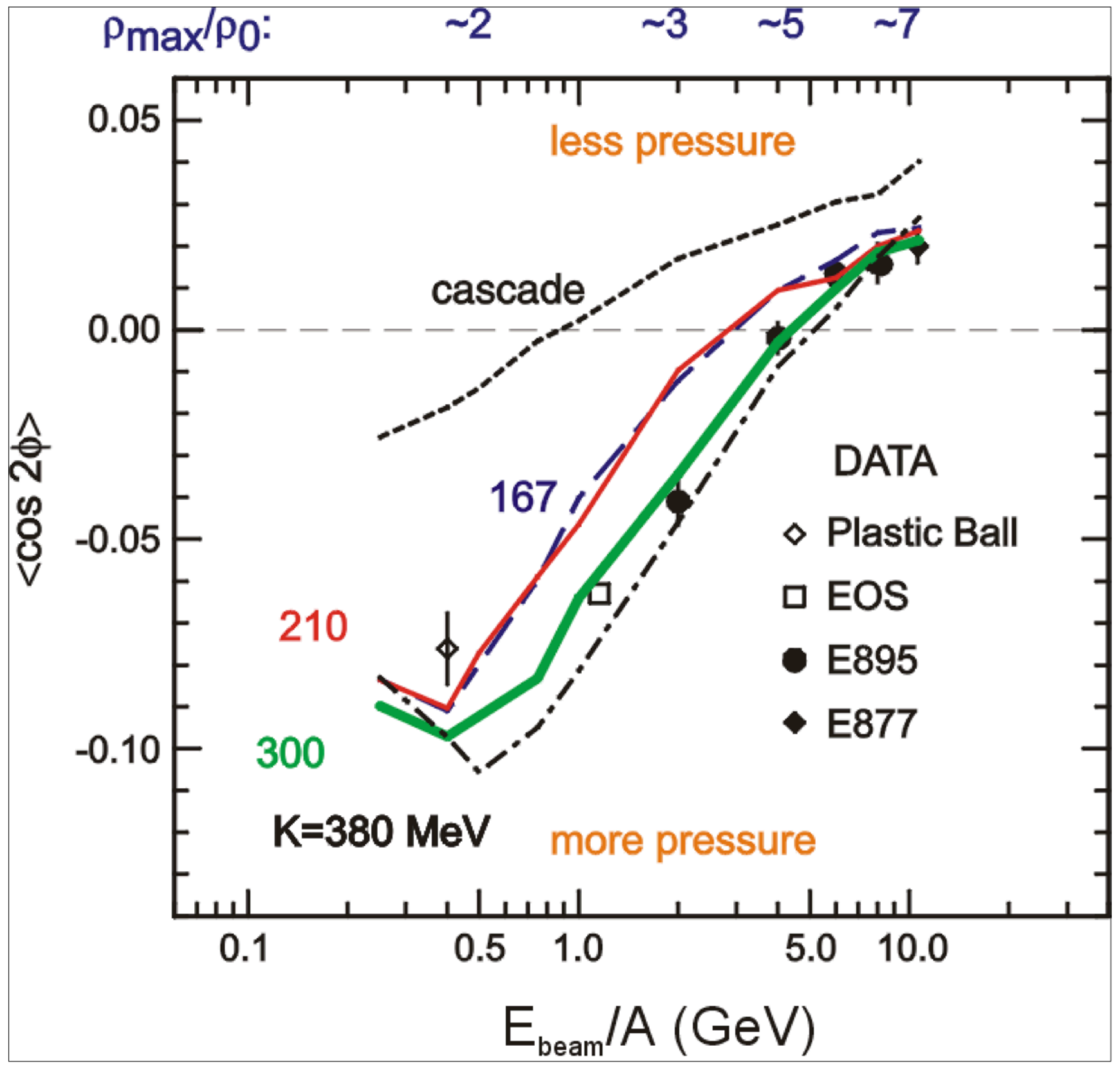
The most common form of the EOS is the "Skyrme potential": $U(n_B) = A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau}$



$$F = \left.\frac{d\langle p_x/A\rangle}{d(y/y_{\rm cm})}\right|_{y/y_{\rm cm}=1} \sim \frac{dv_1}{dy}$$



v_2

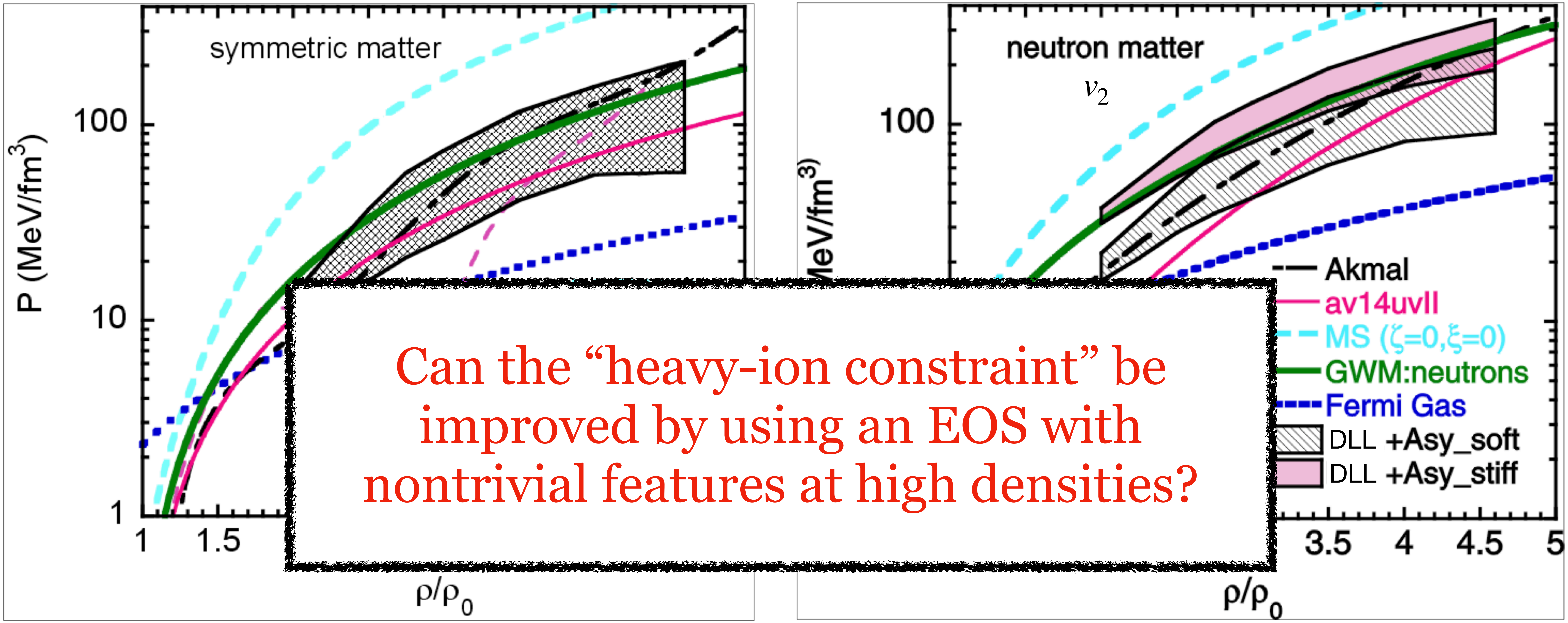


P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

Standard way of modeling the EOS: Skyrme potential

P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

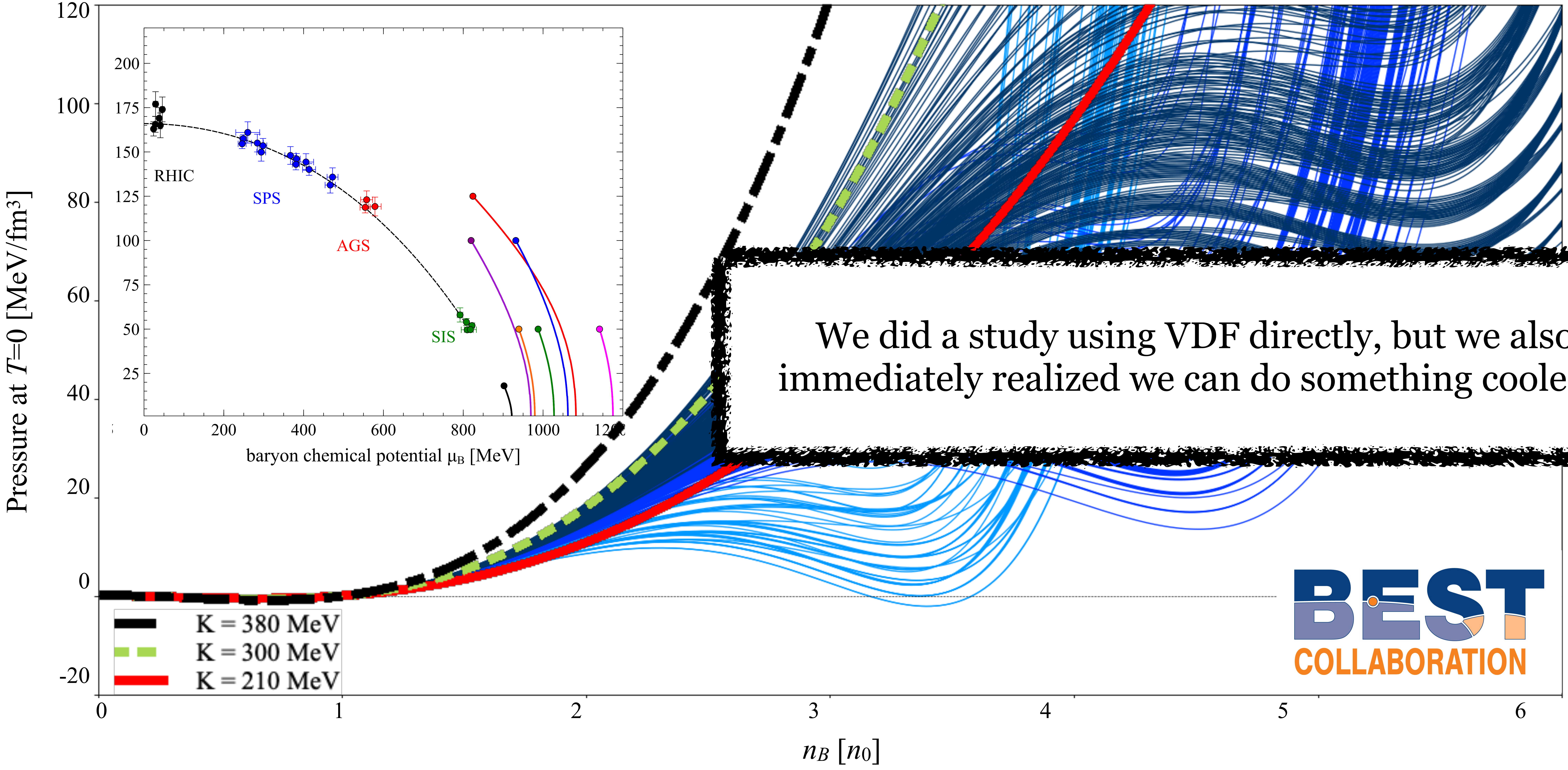
“the heavy-ion constraint”



Can the “heavy-ion constraint” be improved by using an EOS with nontrivial features at high densities?

VDF model: relativistic potentials with two 1st order phase transitions

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

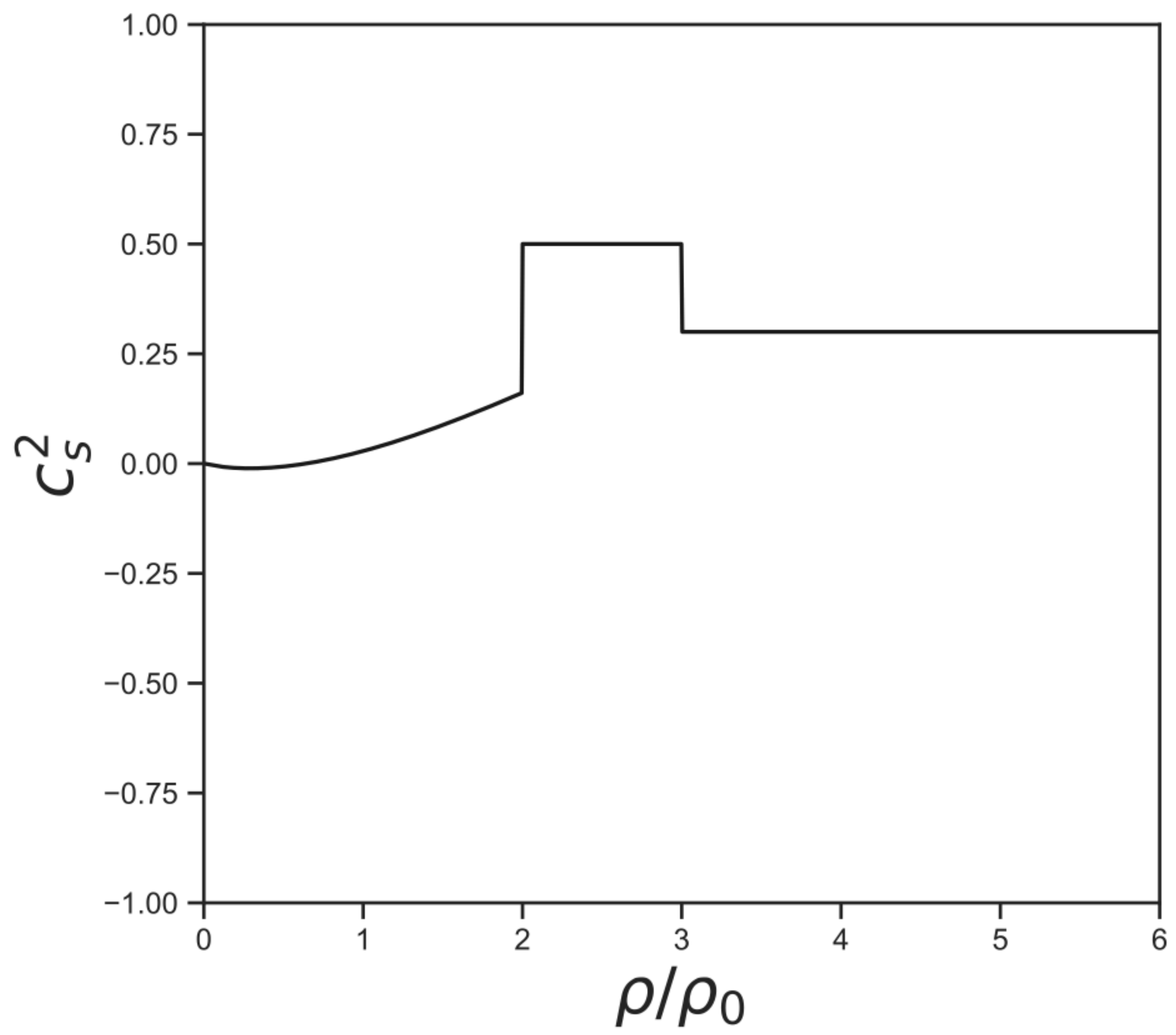


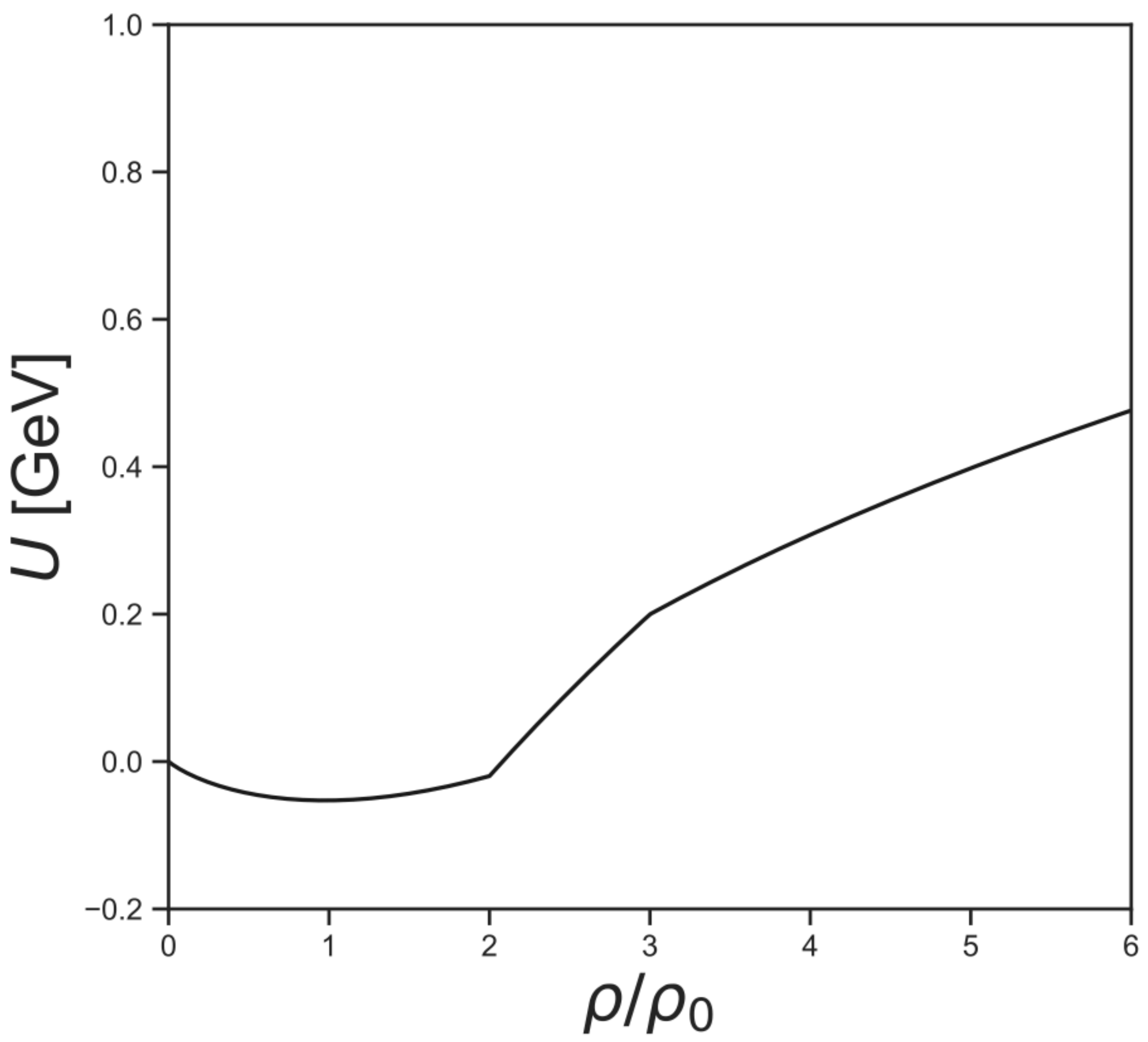
We did a study using VDF directly, but we also immediately realized we can do something cooler…

Bayesian analysis: piecewise parametrization of c_s^2

Piecewise parametrization of $c_s^2(n_B)$:

$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$





1-to-1 relation to the single-particle potential $U(n_B)$:

$$U(n_B) = \begin{cases} U_{\text{Sk}}(n_B) & n_B < n_1 = 2n_0 \\ U_1(n_B) & n_1 < n_B < n_2 \\ \dots & \\ U_k(n_B) & n_k < n_B < n_{k+1} \end{cases}$$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

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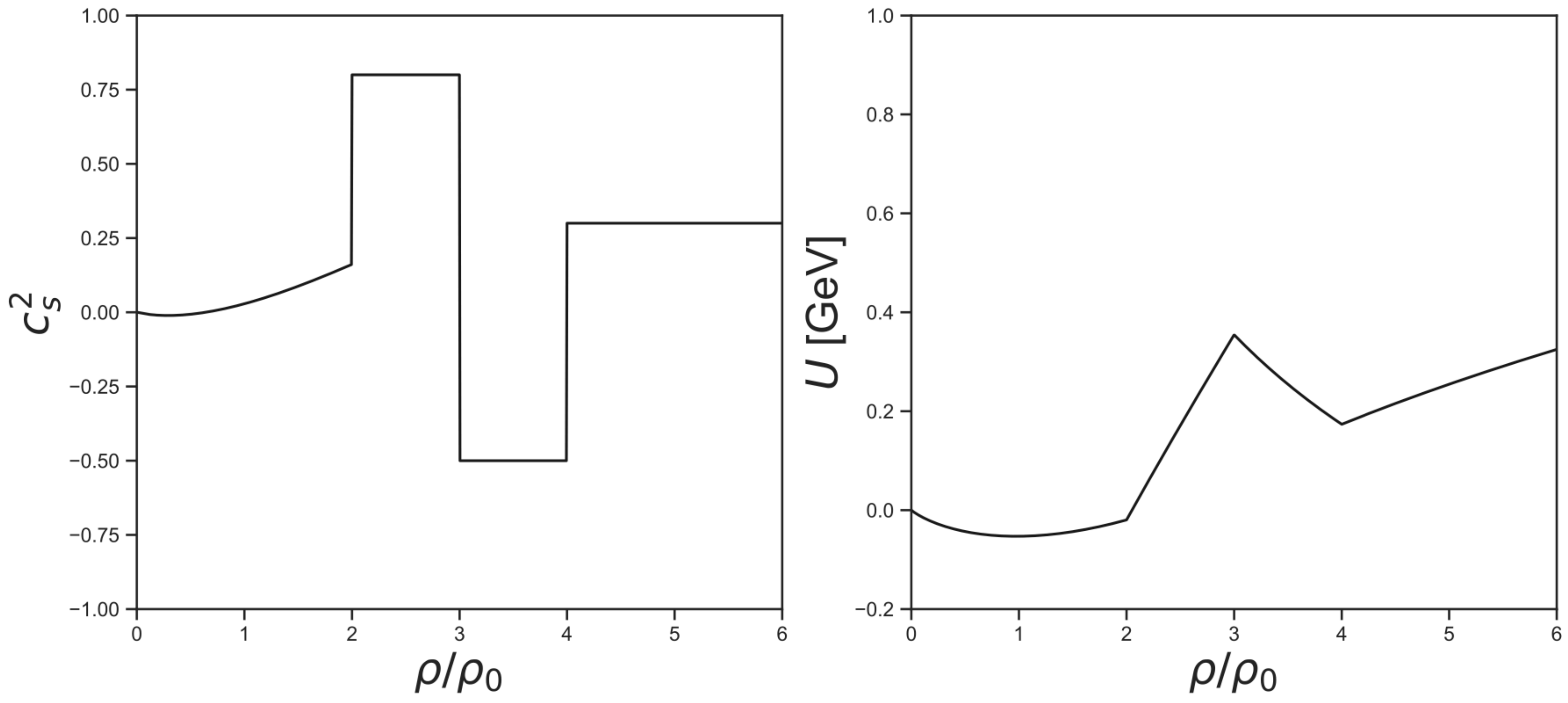

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Gradients of $U(n_B)$ enter the EOMs
= directly affect the evolution in simulations

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
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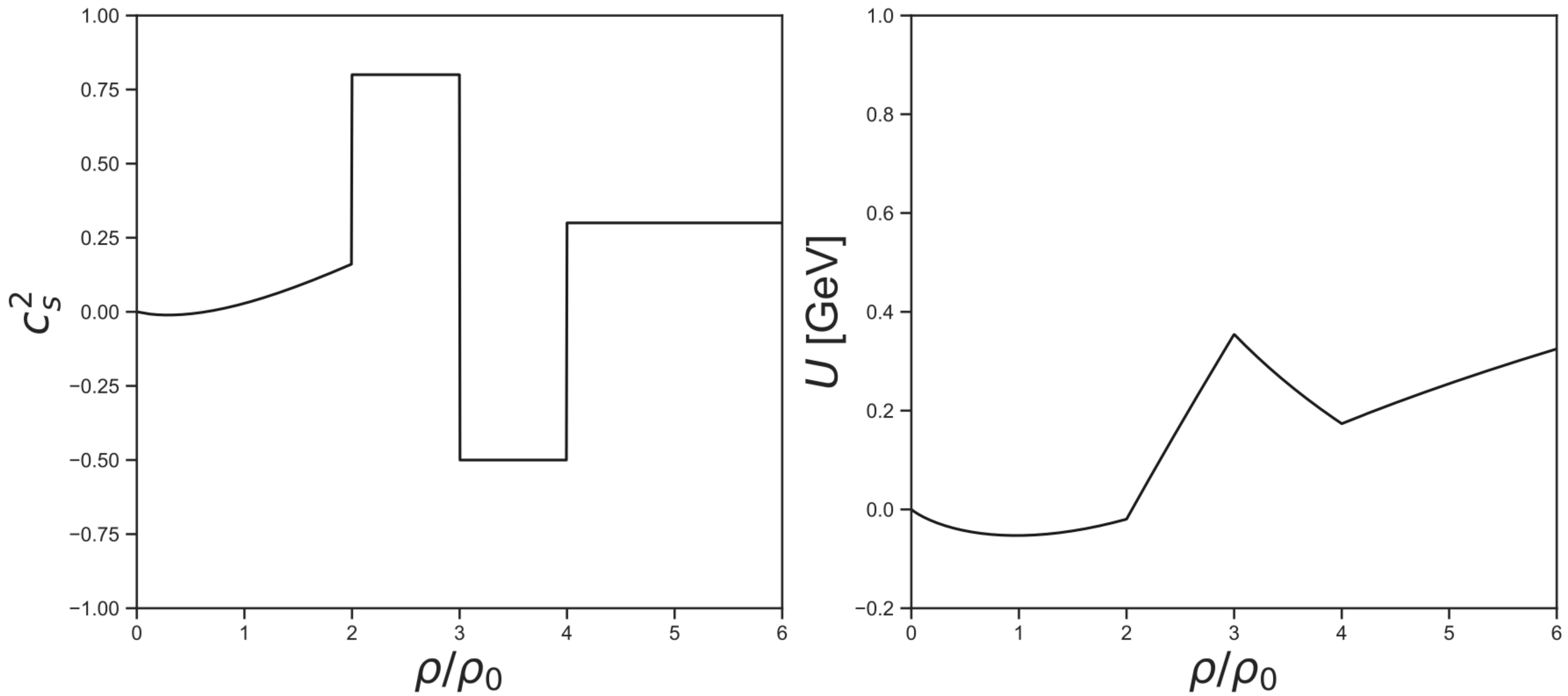

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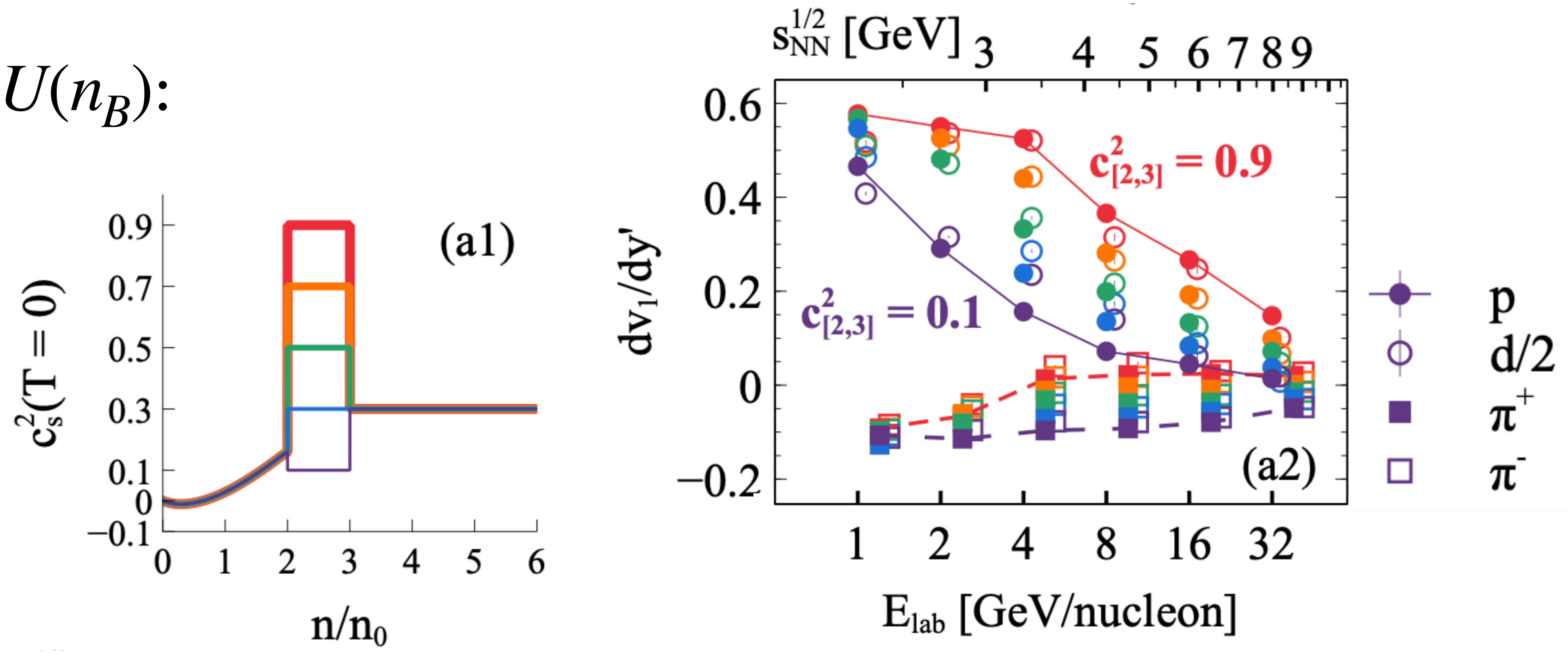
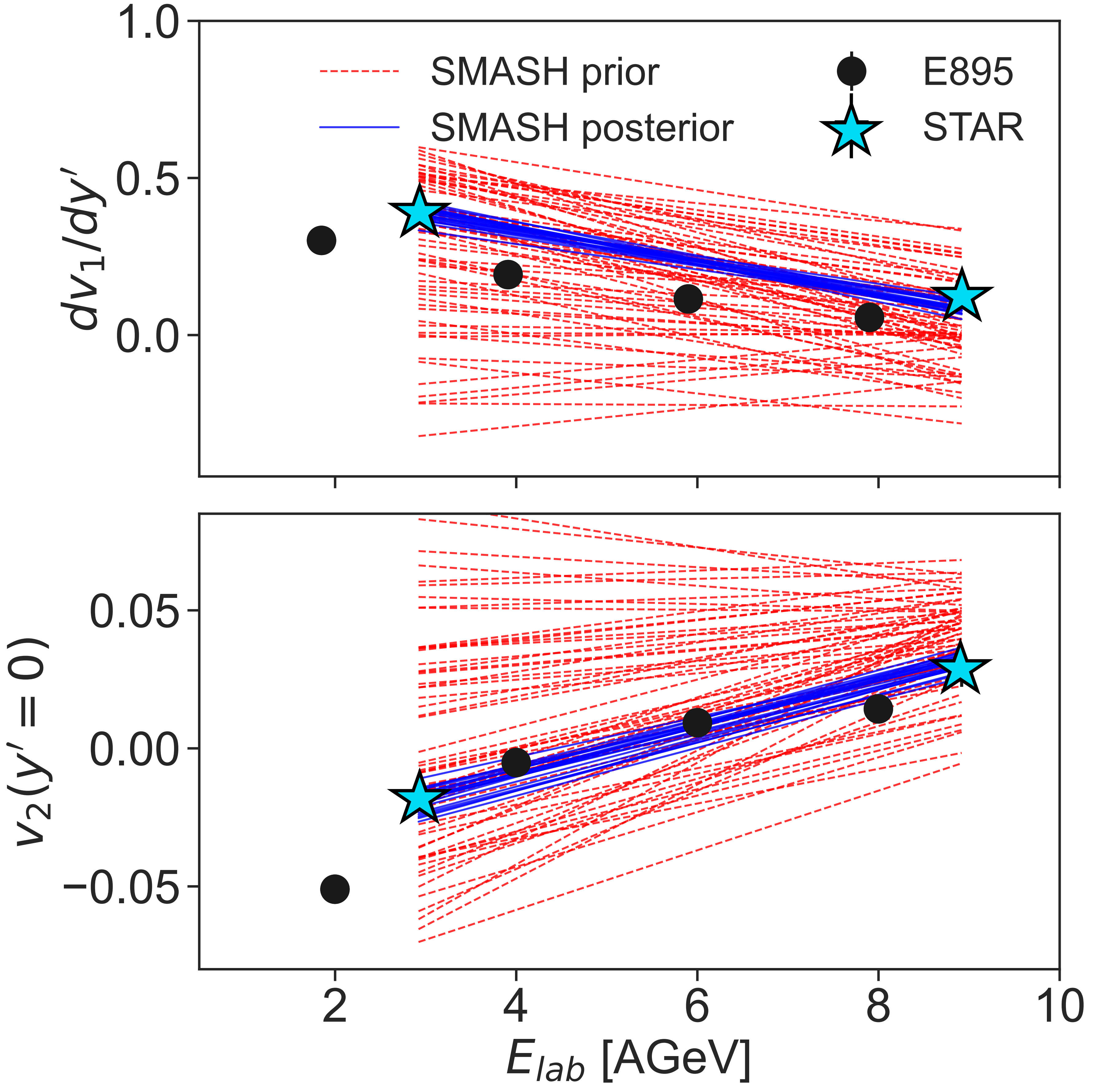
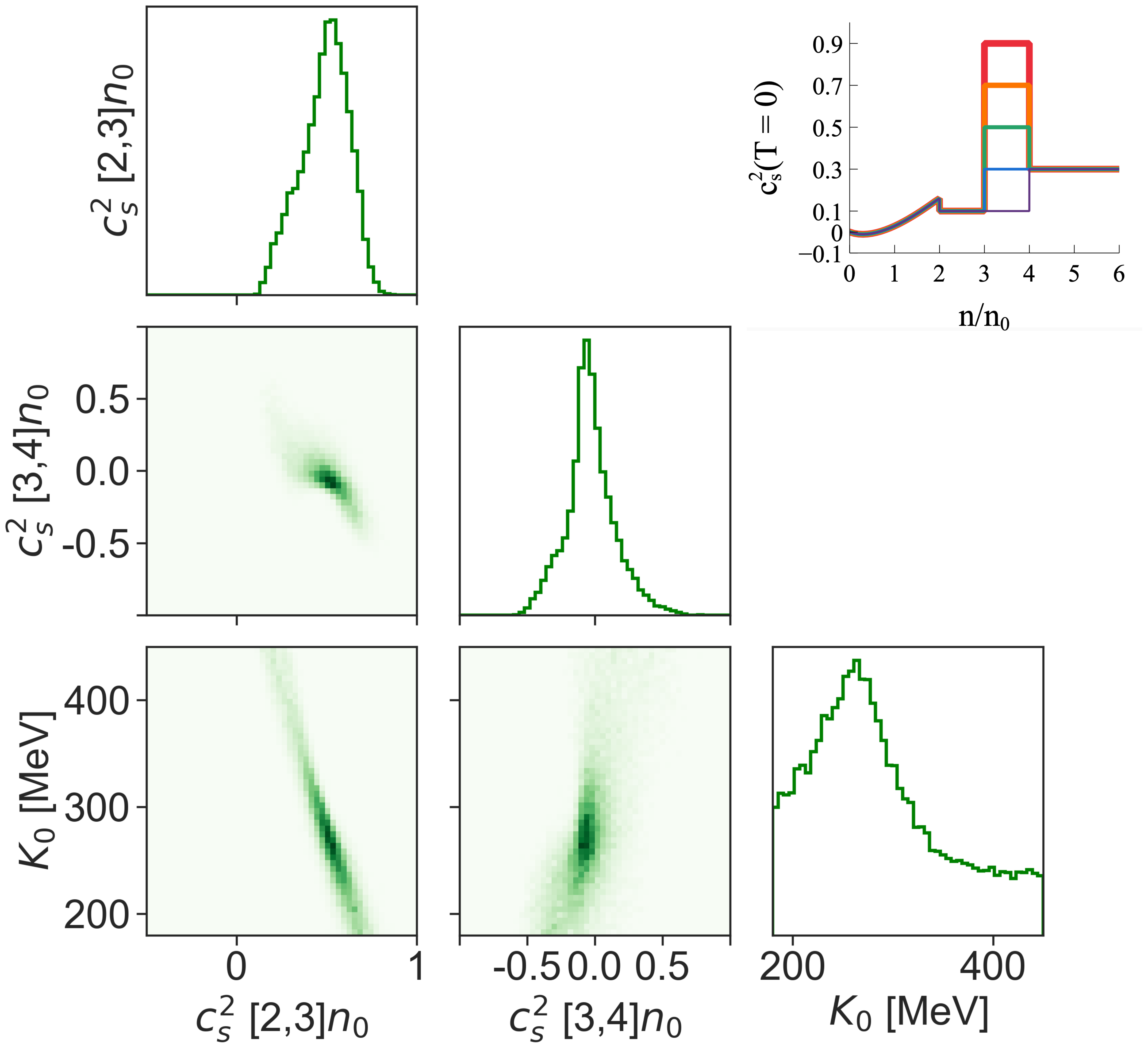


D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

Bayesian analysis of BES flow in BUU with varying K_0, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$



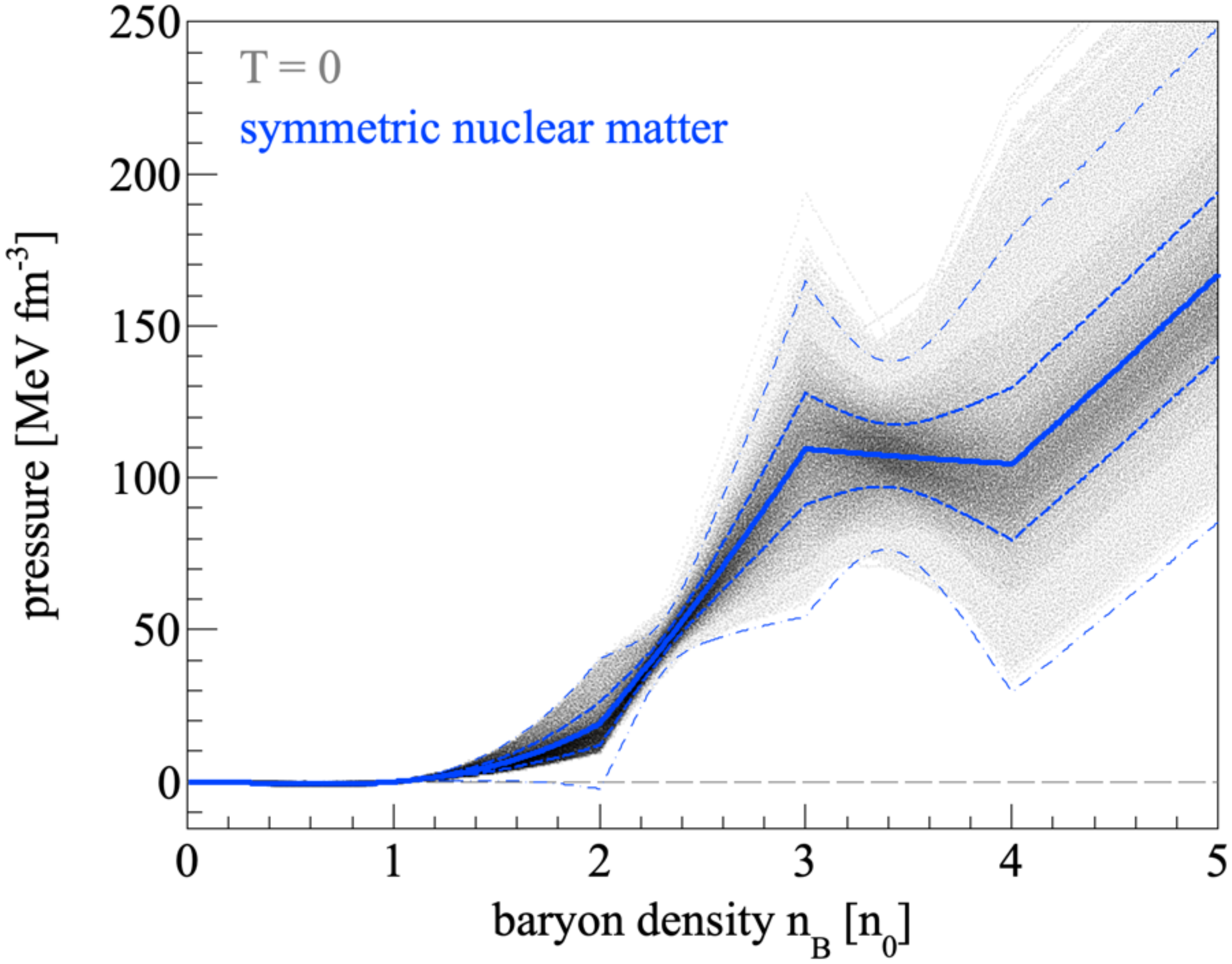


The maximum a posteriori probability (MAP) parameters are

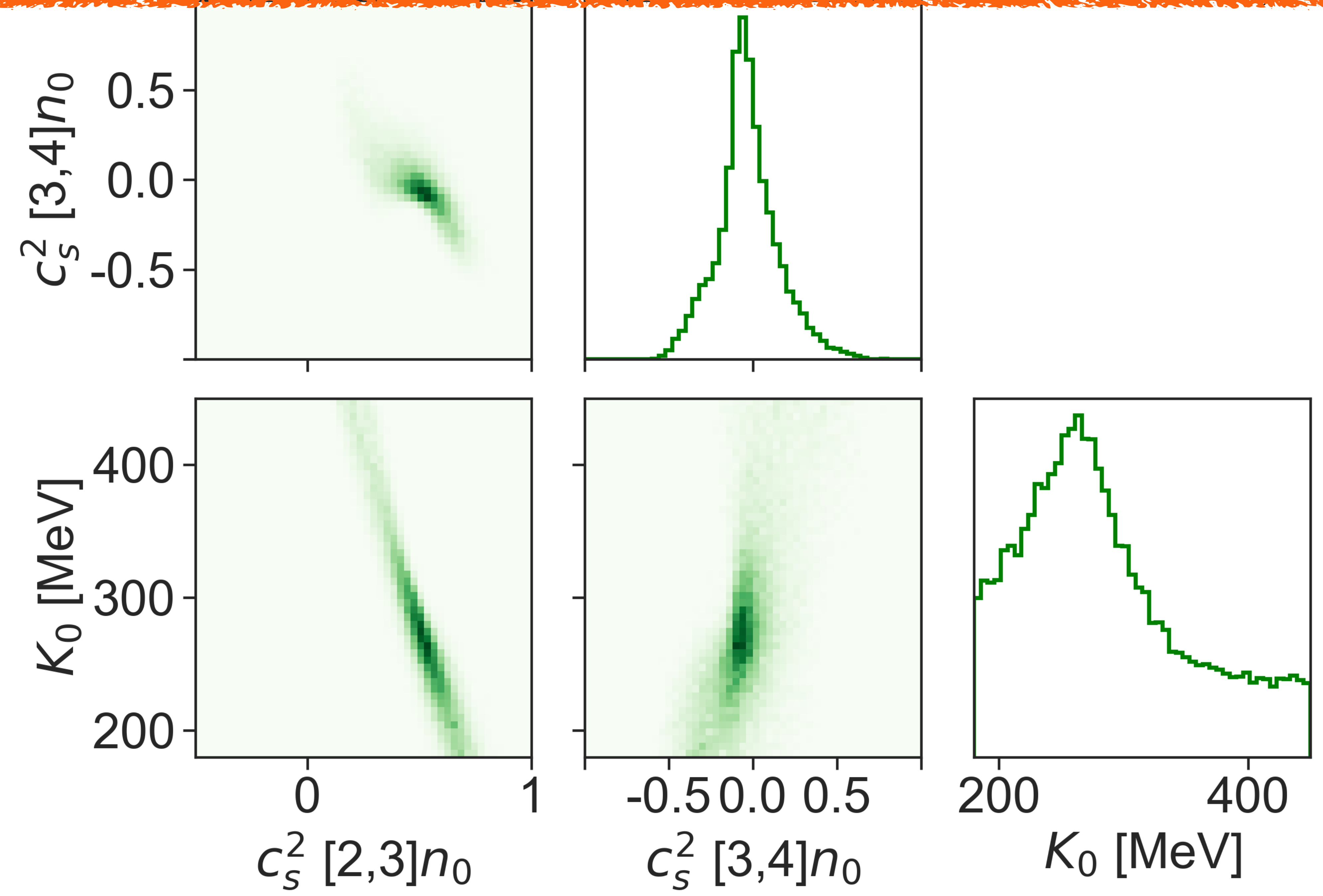
$K_0 = 285 \pm 67$ MeV, $\quad c^2_{[2,3]n_0} = 0.49 \pm 0.13, \quad c^2_{[3,4]n_0} = -0.03 \pm 0.15$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

Bayesian analysis of BES flow in BUU with varying K_0, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$



The constrained EOS is very stiff at $n_B \in (2,3)n_0$ and very soft at $n_B \in (3,4)n_0$



The maximum a posteriori probability (MAP) parameters are

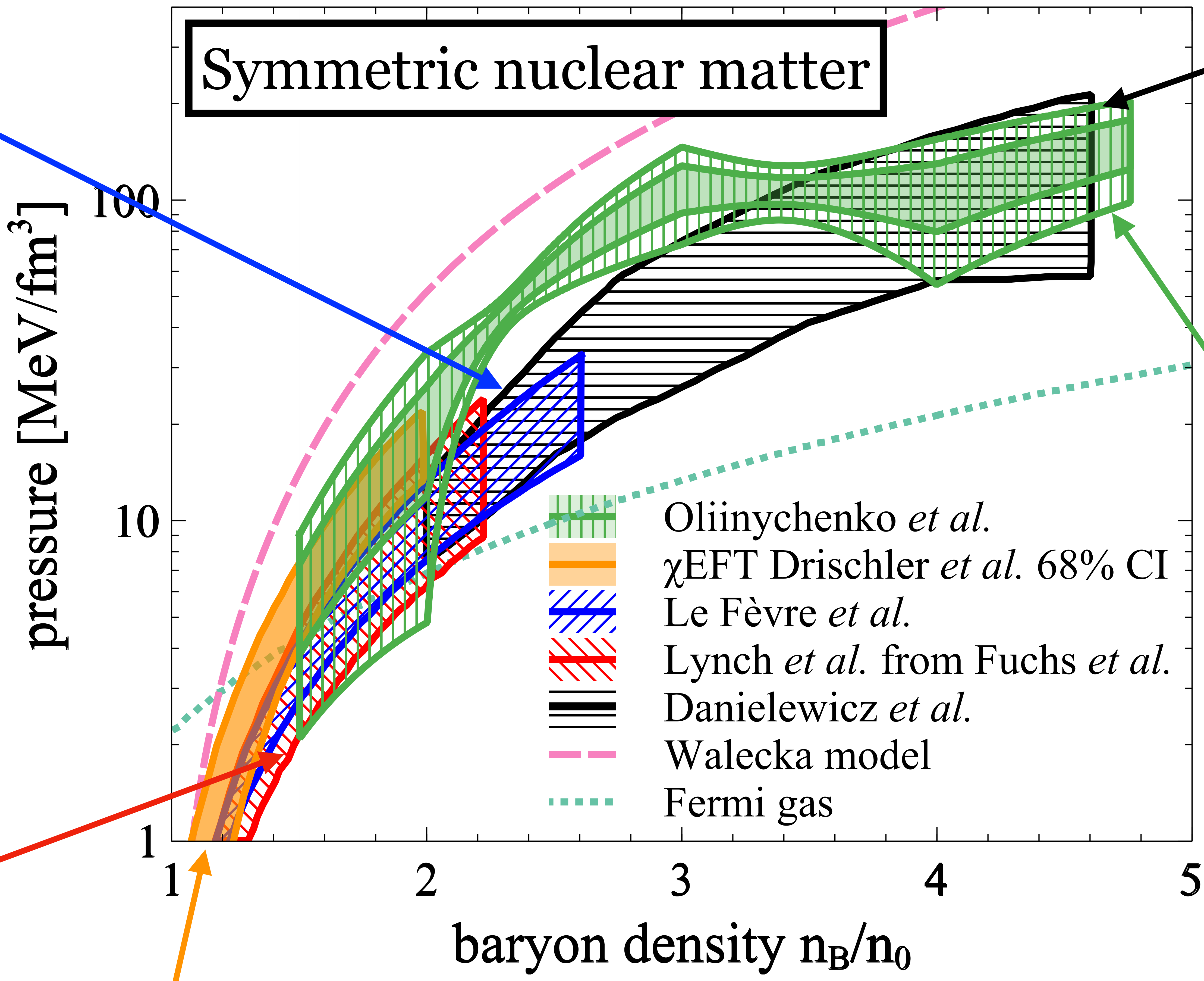
$K_0 = 285 \pm 67$ MeV, $\quad c^2_{[2,3]n_0} = 0.49 \pm 0.13, \quad c^2_{[3,4]n_0} = -0.03 \pm 0.15$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

EOS of symmetric nuclear matter: selected (*few*) results

197Au+197Au @ 0.4—1.5 GeV/u
($\sqrt{s_{NN}}$ = 2.07 — 2.52 GeV)
observables: proton flow (FOPI)
model used: **isospin QMD (IQMD)** w/ nucleons, Δ, N*(1440), deuterons, tritons;
EOS parametrized by K_0;
momentum dependence
A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, C. Hartnack, Nucl. Phys. A 945, 112 (2016), arXiv:1501.05246

197Au+197Au & 12C+12C @ < 1.5 GeV/u
($\sqrt{s_{NN}}$ < 2.5 GeV)
observables: subthreshold kaon production (KaoS)
model used: **QMD** w/ nucleons, Δ, N*(1440), pions, kaons;
EOS parametrized by K_0;
kaon potentials, momentum dependence
C. Fuchs *et al.*, Prog. Part. Nucl. Phys. **53**, 113–124 (2004) arXiv:nucl-th/0312052



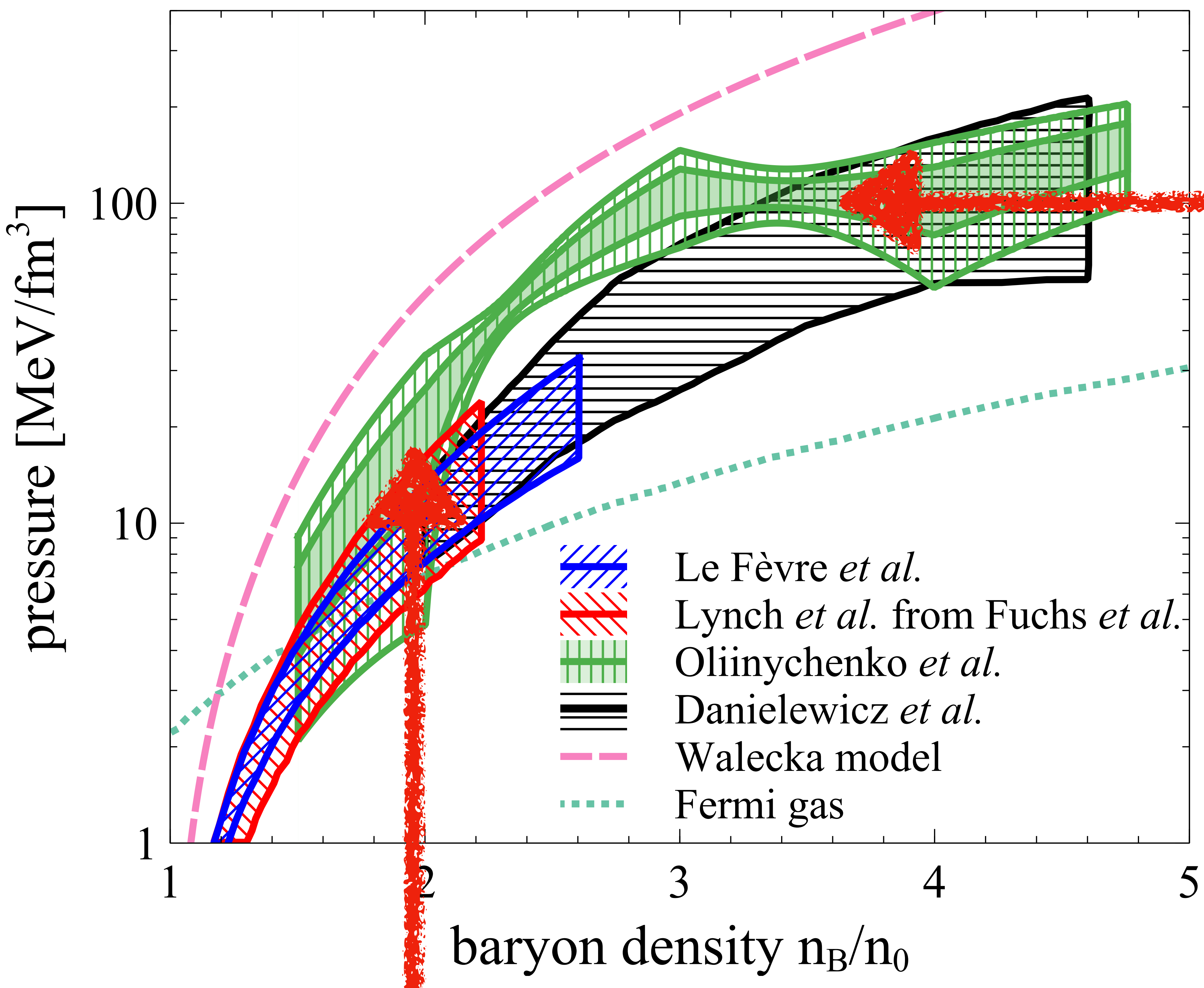
A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024) arXiv:2301.13253

χEFT
C. Drischler *et al.*, Phys. Rev. C **102** 5, 054315 (2020) arXiv:2004.07805

197Au+197Au @ 0.15—10 GeV/u
($\sqrt{s_{NN}}$ = 1.95 — 4.72 GeV)
observables: proton flow (Plastic Ball, EOS, E877, E895)
model used: **pBUU** w/ nucleons, Δ, N*(1440), pions;
EOS parametrized by K_0;
momentum dependence
P. Danielewicz, R. Lacey, W. G. Lynch, Science **298**,1592–1596 (2002)

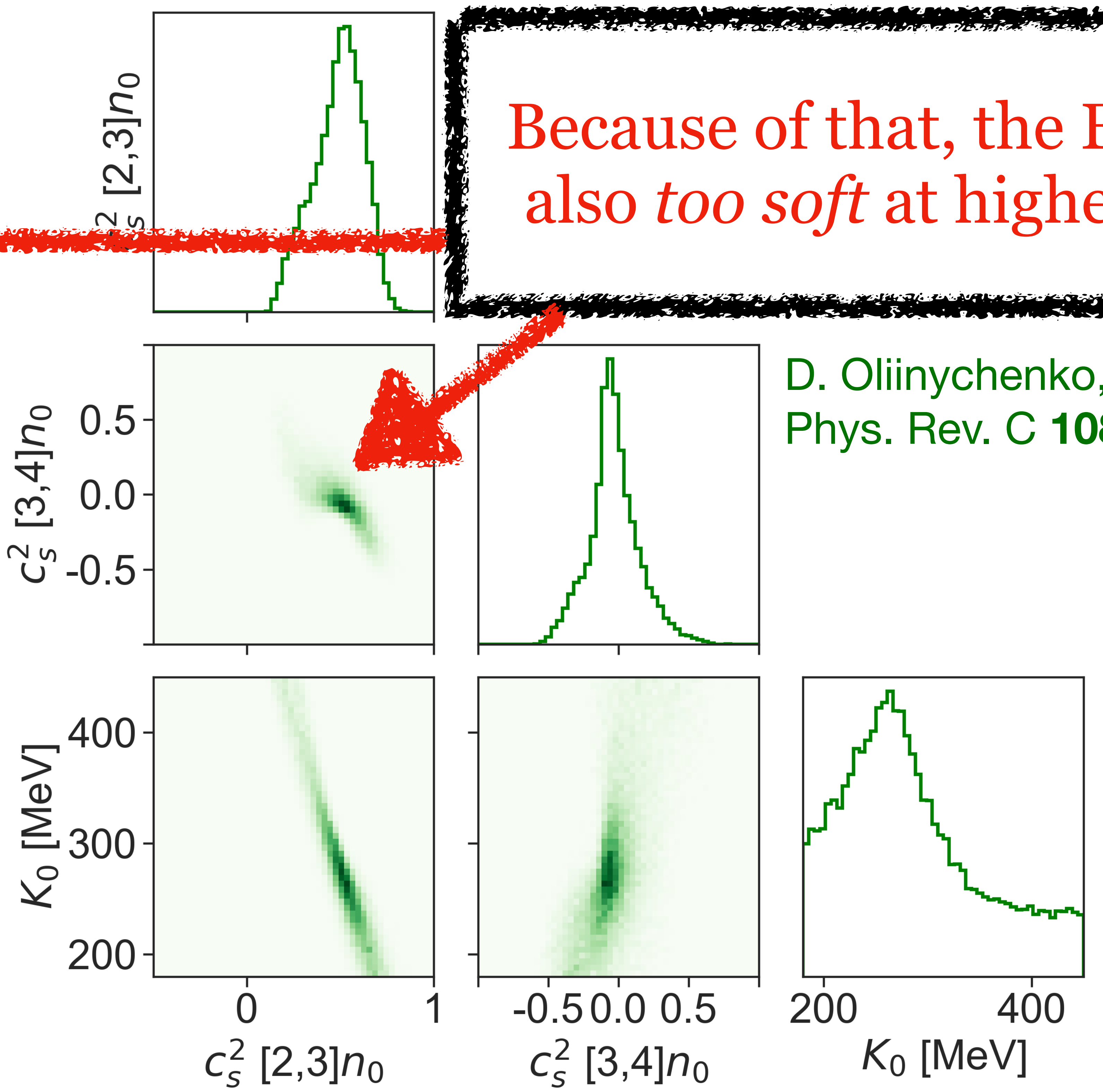
197Au+197Au @ 2.9—9 GeV/u
($\sqrt{s_{NN}}$ = 3 — 4.5 GeV)
observables: proton flow (STAR)
model used: SMASH w/ over 120 hadronic species, including deuterons;
relativistic EOS parametrized independently in different density regions;
NO momentum dependence
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

Momentum-dependence of nuclear matter interactions



A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024) arXiv:2301.13253

Without momentum dependence, “artificial” additional source of repulsion is needed = the extracted EOS is too stiff?

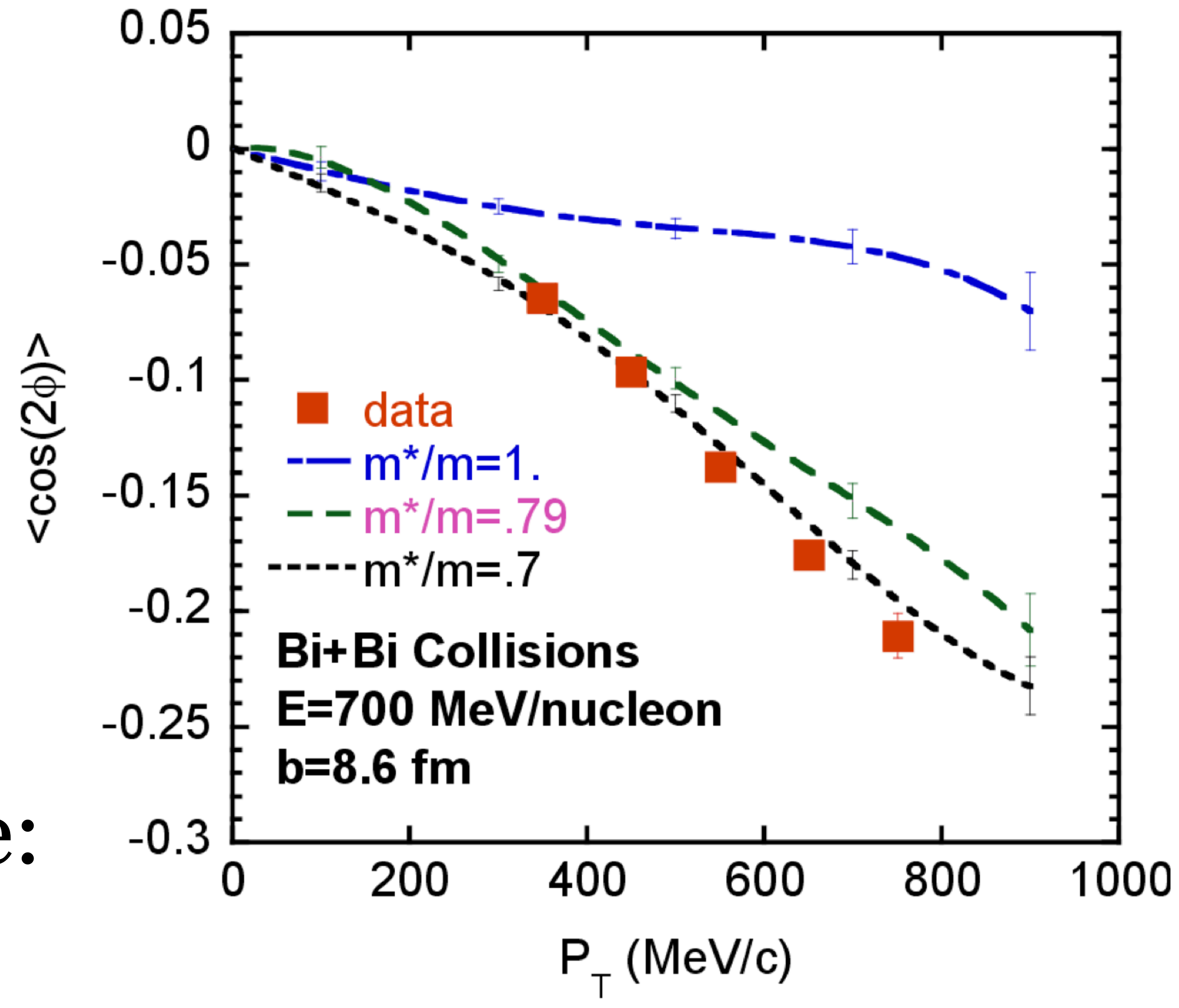


Because of that, the EOS may be also *too soft* at higher densities

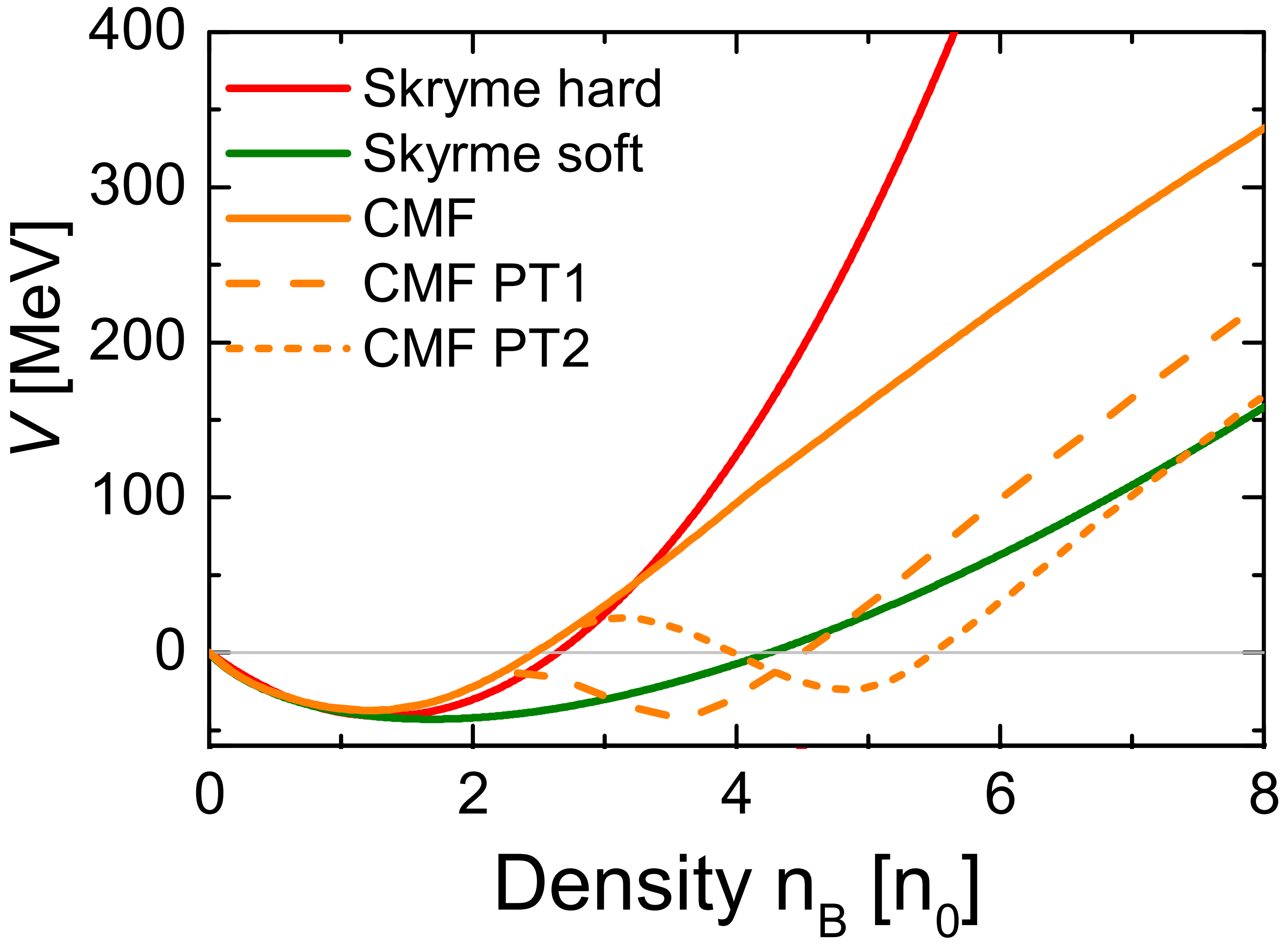
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

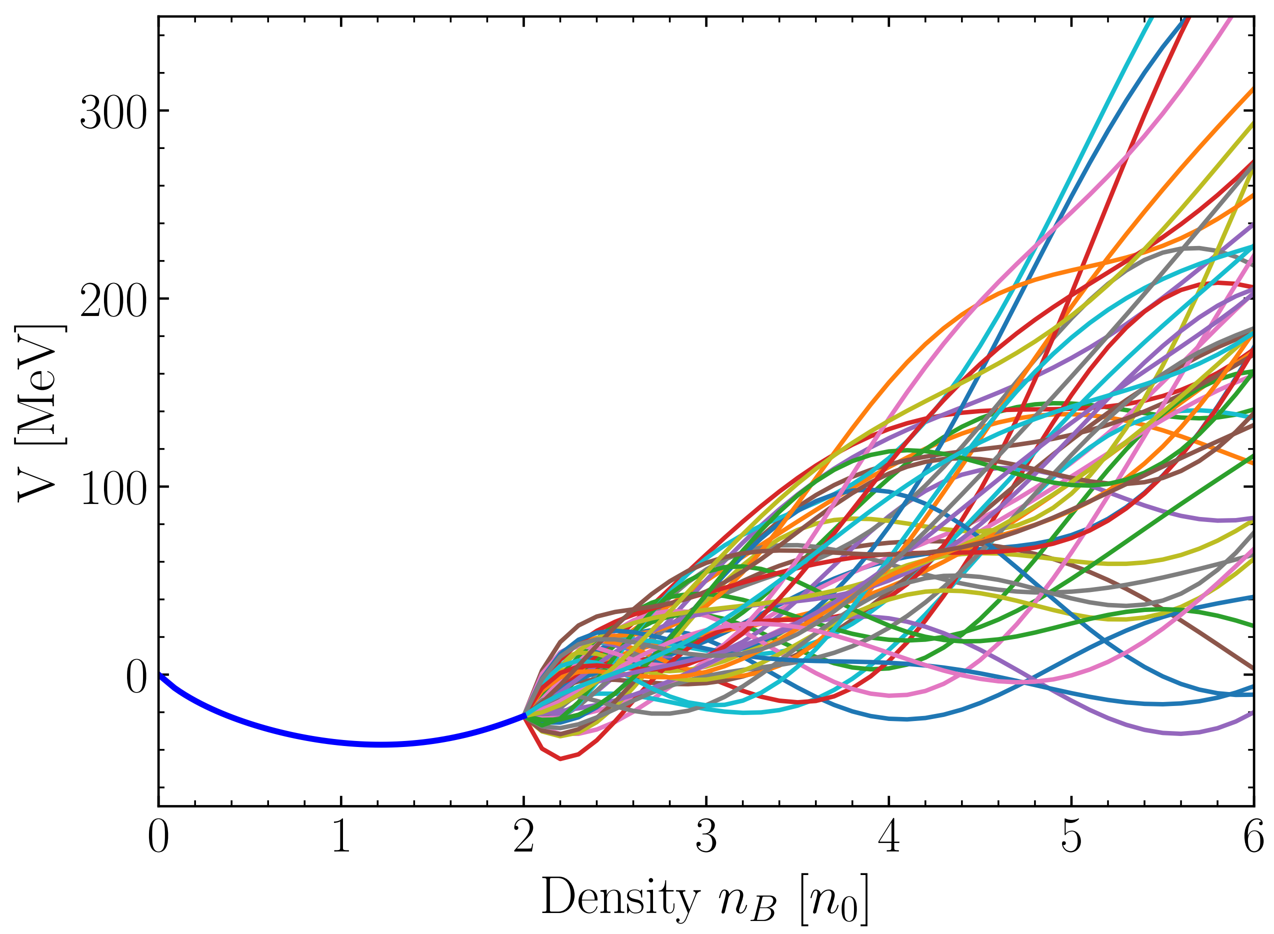
P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:0208016

Use peripheral collisions to constrain the *p*-dependence:



Bayesian analysis of flow data in UrQMD





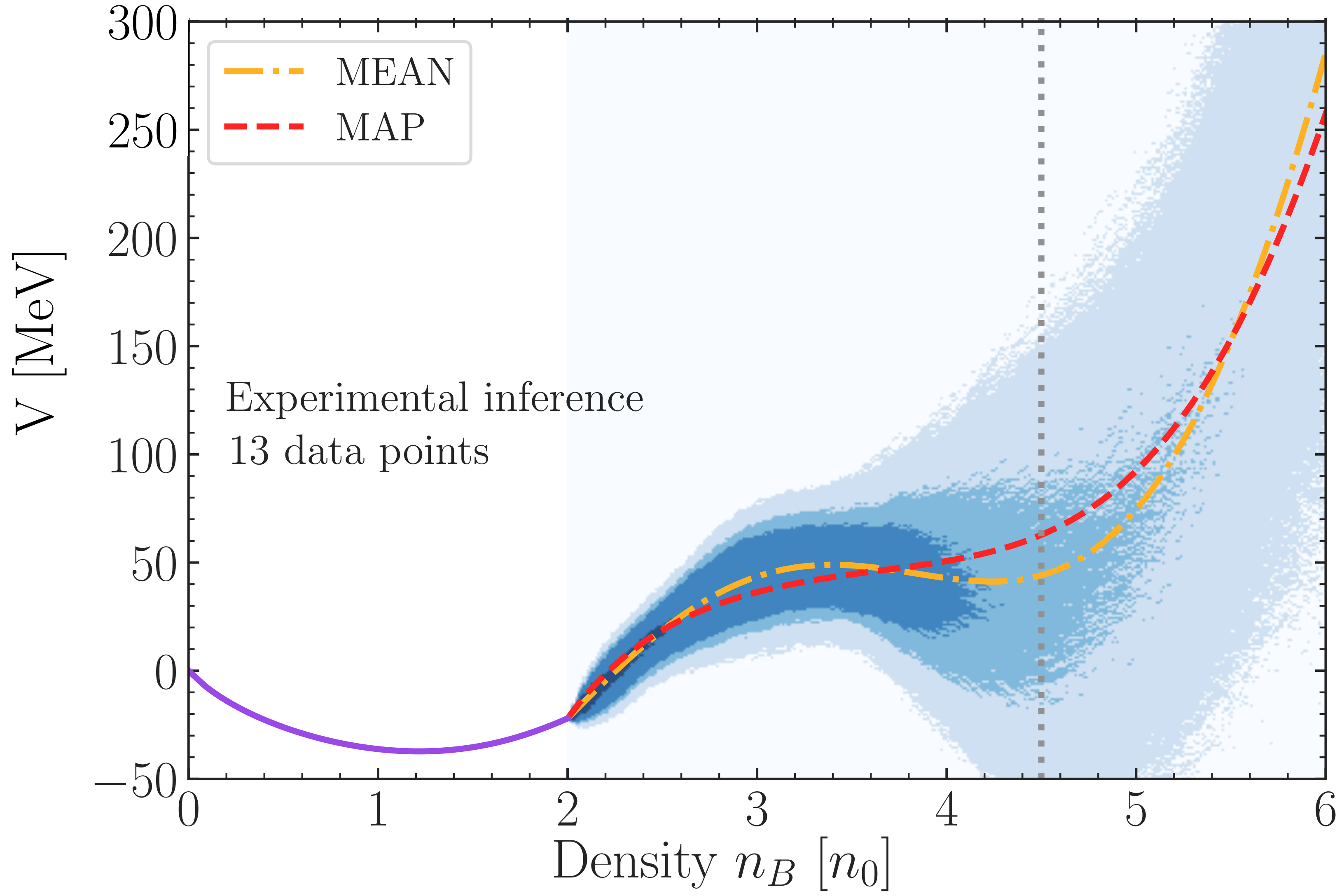
M. Omana Kuttan, J. Steinheimer, K. Zhou, H. Stoecker,
Phys. Rev. Lett. **131** 20, 202303 (2023)
arXiv:2211.11670

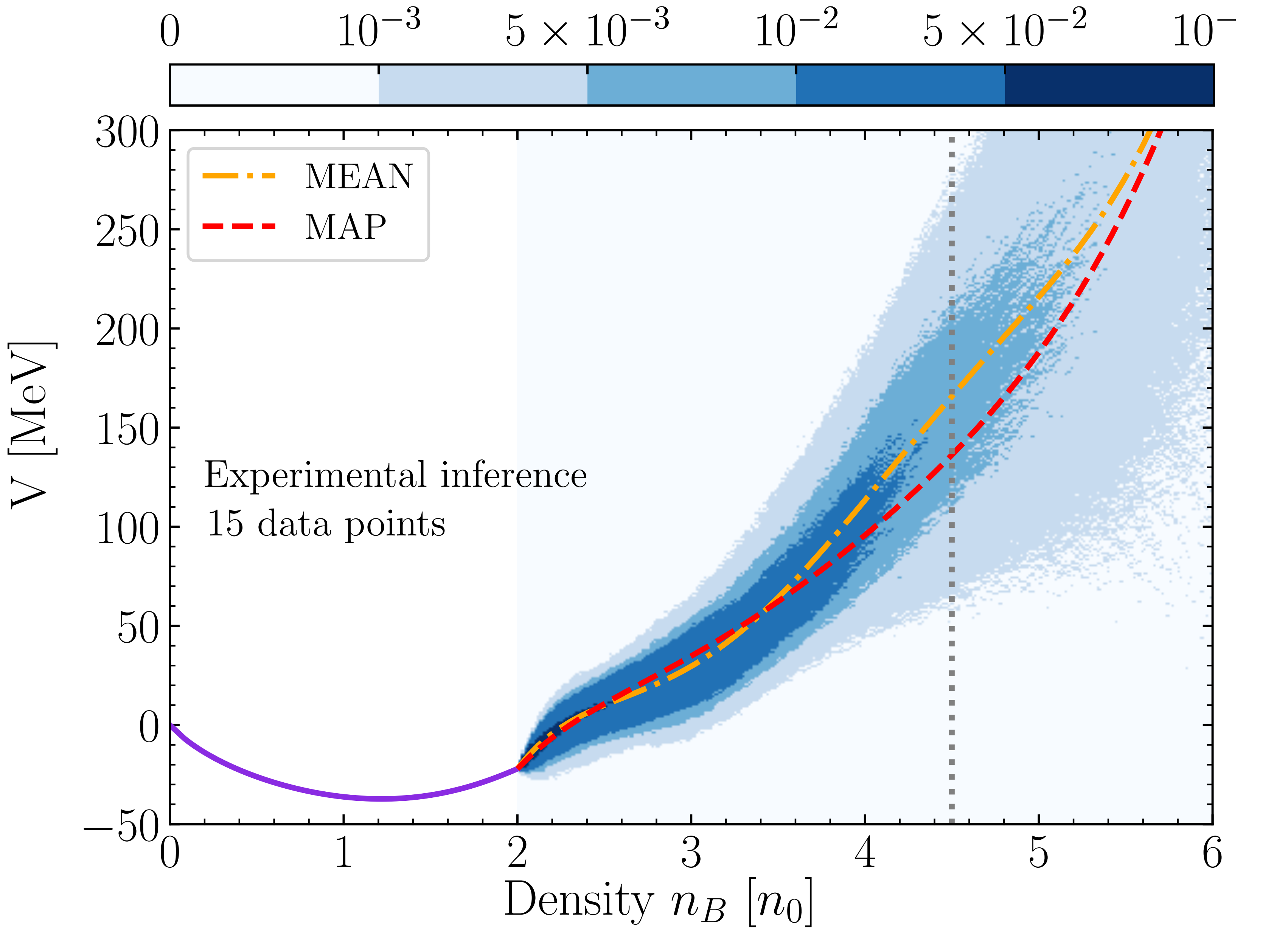
$$V(n_B) = \begin{cases} V_{\text{CMF}} & n_B \leq 2n_0 \\ \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 1\right)^i + C & n_B > 2n_0 \end{cases}$$

proton mean transverse kinetic energy $\langle m_T \rangle - m_0$:
$\sqrt{s_{\text{NN}}} \in [3.83,\ 8.86]$ GeV

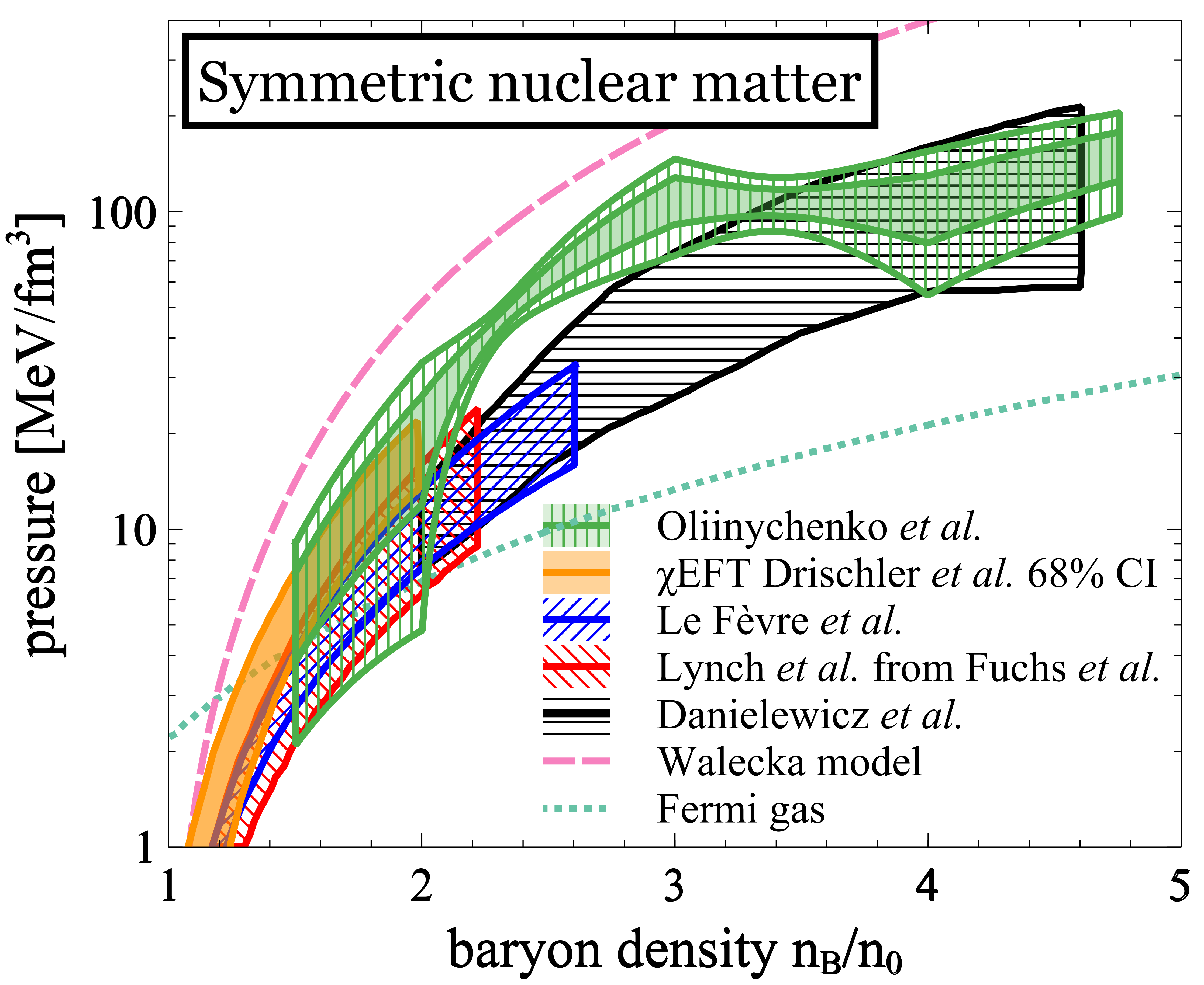
proton elliptic flow v_2 at midrapidity:
$\sqrt{s_{\text{NN}}} \in [2.24,\ 4.72]$ GeV

13 points = excluding $\langle m_T \rangle - m_0$
at the two lowest collision energies
$\sqrt{s_{\text{NN}}} = 3.83,\ 4.29$ GeV

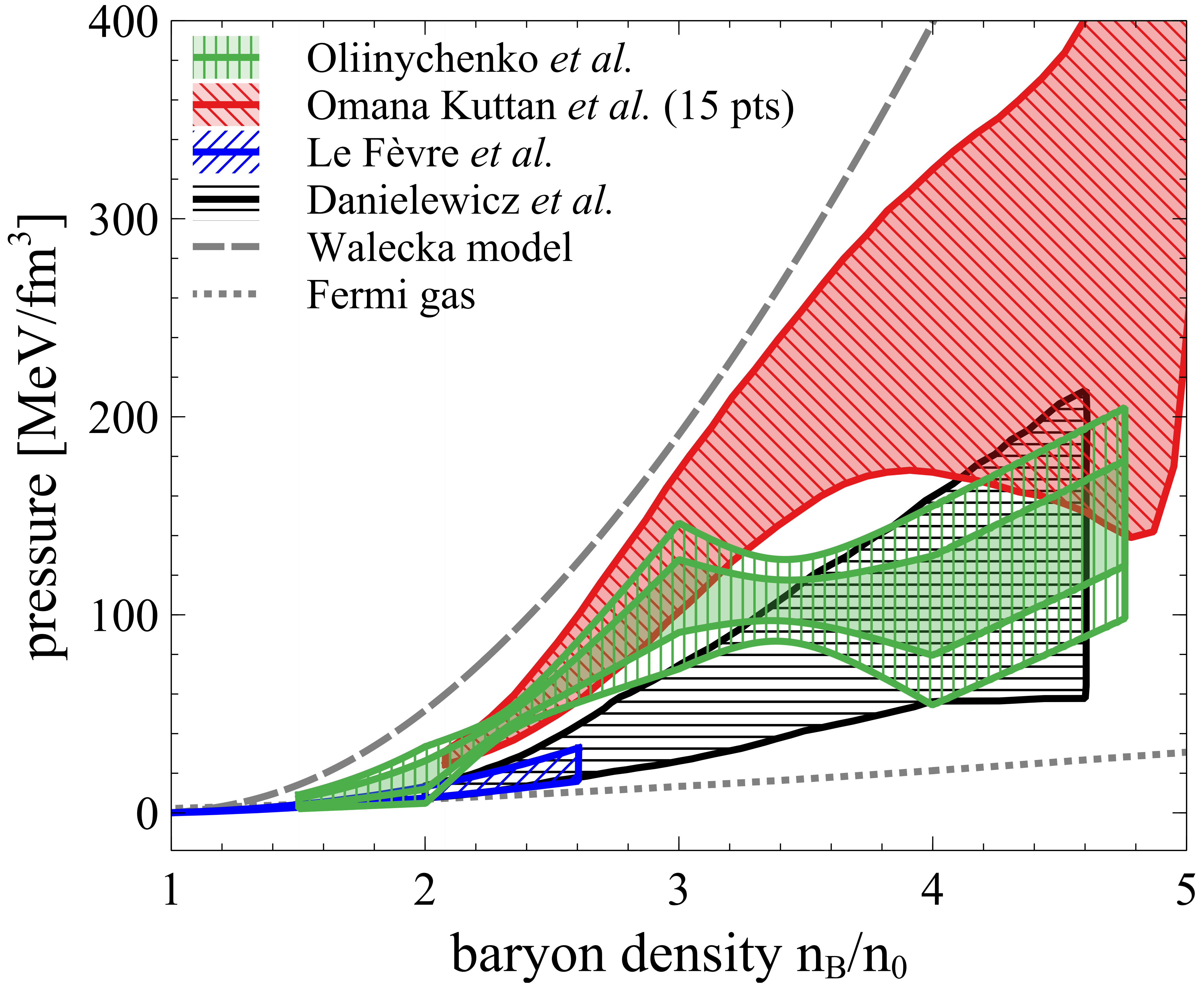




EOS of symmetric nuclear matter: selected (*few*) results

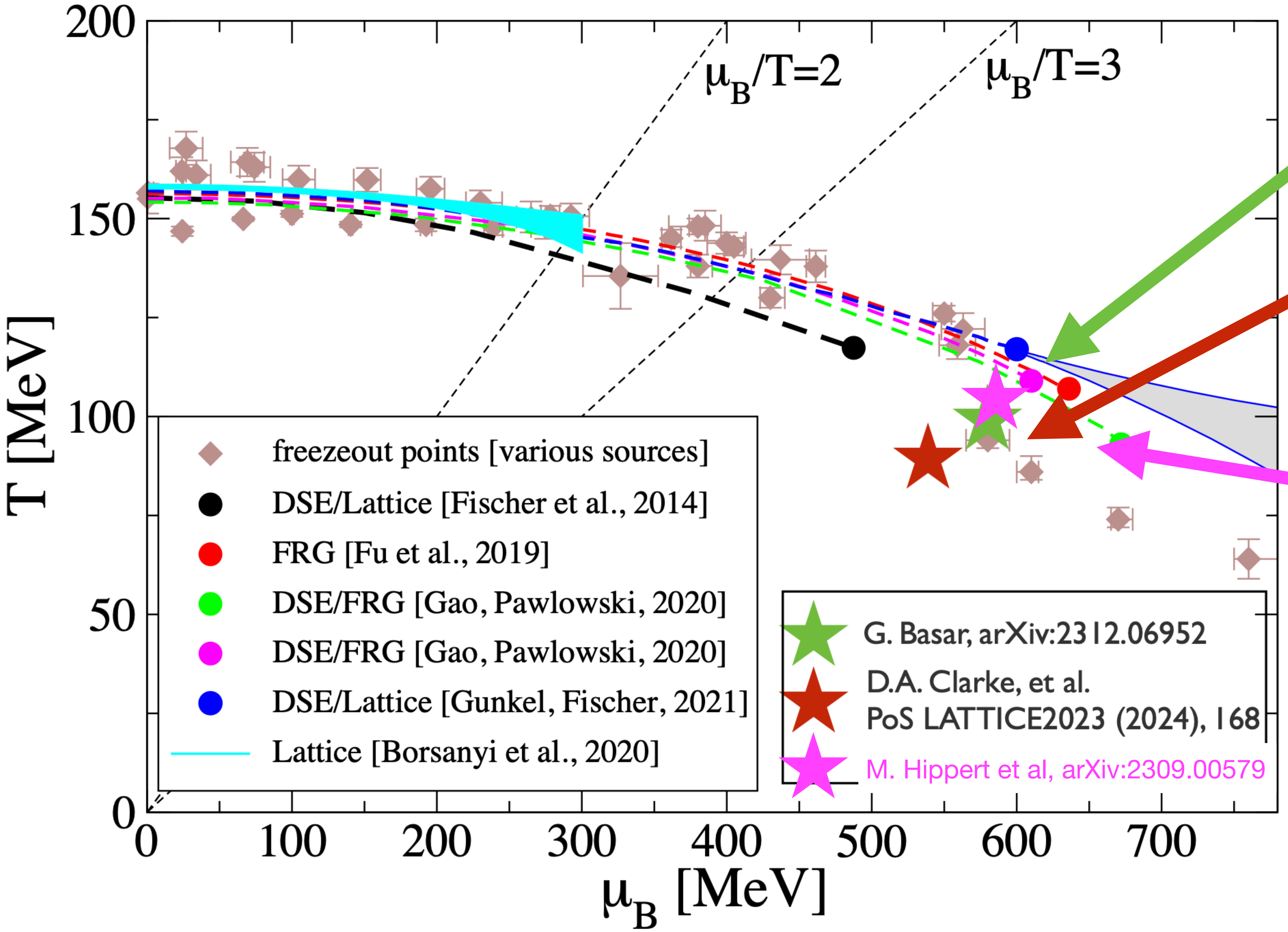


A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024)
arXiv:2301.13253



L. Du, **A. Sorensen**, M. Stephanov, Int. J. Mod. Phys. E
(available online), arXiv: 2402.10183

The QCD critical point: recent theoretical developments



tracing Yang-Lee edge singularities with:

imaginary baryon chemical potential

G. Basar, Phys. Rev. C **110**, 015203 (2024), arXiv:2312.06952

Taylor series expansion coefficients

D. A. Clarke *et al.*, arXiv:2405.10196

holography w/ LQCD EOS at $\mu_B = 0$

M. Hippert *et al.*, arXiv:2309.00579

Dramatic change for the field:
we need *evidence that withstands comparison with theory*

The QCD CP from finite-size scaling: universal behavior

Near CP:

$$c_\infty(t,0) \sim |t|^{-\alpha}$$
$$\tilde{n}_\infty(t,0) \sim (-t)^{\beta}$$
$$\tilde{n}_\infty(0,m) \sim m^{\frac{1}{\delta}}$$
$$\chi_\infty(t,0) \sim |t|^{-\gamma}$$

$$\xi_\infty(t,0) \sim |t|^{-\nu}$$
$$\xi_\infty(0,m) \sim |m|^{-\nu_c}$$

$$t \equiv \frac{T - T_c}{T_c}$$
$$m \equiv \frac{\mu - \mu_c}{\mu_c}$$

For a thermodynamic quantity $X \sim |t|^{-\sigma}$: $\quad X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}}$

Scaling is not unique to critical phenomena, e.g., Kepler's third law!
The orbital period of a planet scales as the cube of the semi-major axis of its orbit:

$$P^2 = a^3$$

The important question for scaling is: what is the scale relevant to the problem?

The QCD CP from finite-size scaling: universal behavior

Near CP:

$$c_\infty(t,0) \sim |t|^{-\alpha} \qquad \xi_\infty(t,0) \sim |t|^{-\nu}$$

$$\tilde{n}_\infty(t,0) \sim (-t)^{\beta} \qquad \xi_\infty(0,m) \sim |m|^{-\nu_c}$$

$$\tilde{n}_\infty(0,m) \sim m^{\frac{1}{\delta}}$$

$$\chi_\infty(t,0) \sim |t|^{-\gamma}$$

$$t \equiv \frac{T - T_c}{T_c}$$

$$m \equiv \frac{\mu - \mu_c}{\mu_c}$$

For a thermodynamic quantity $X \sim |t|^{-\sigma}$: $\quad X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}}$

CP: infinite volume concept

In real world ξ does not go to infinity = thermodynamic functions do not exhibit singularities

ξ is bound by the size of the system L $\quad \Rightarrow \quad X_L(t_L) \sim L^{\frac{\sigma}{\nu}}$

$$\Rightarrow \quad X_L(t_L) = L^{\frac{\sigma}{\nu}}\phi(t,L) = L^{\frac{\sigma}{\nu}}\phi(tL^{\frac{1}{\nu}})$$

$$\Rightarrow \quad X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

one can find CP by plotting

Finite size vs. window size

$$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

Finite-size scaling (original): change the size of the system, calculate $X_L(t_L)$, repeat

However: changing SIZE is not always possible or doesn't probe the same system (bird flocks, heavy-ions)

Solution: study the dependence of X on the size of the *subsystem* that is considered

D. Martin, T. Ribeiro, S. Cannas, *et al.*, Box scaling as a proxy of finite size correlations, Sci Rep 11, 15937 (2021)

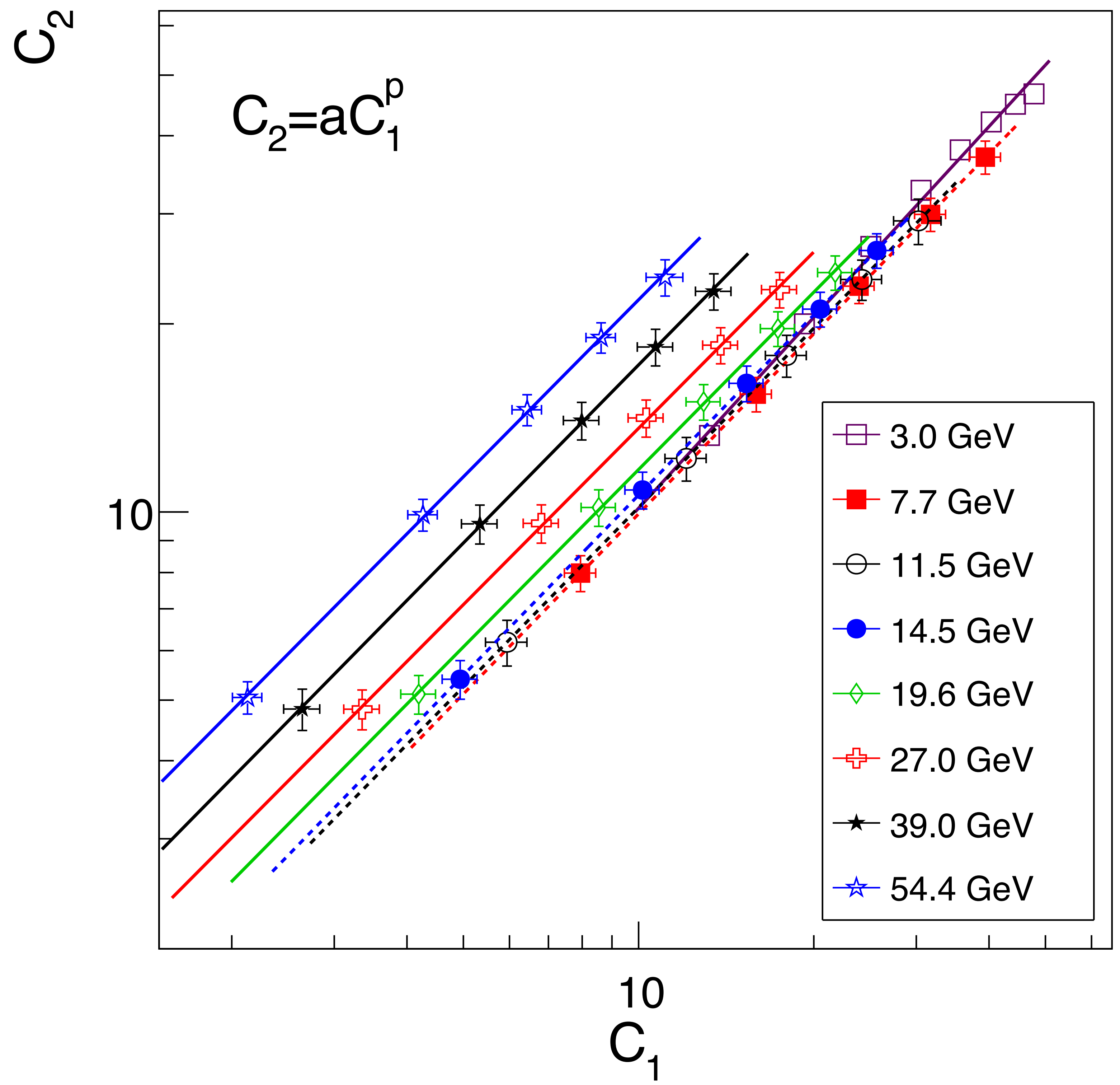
What are the scales relevant to the problem?

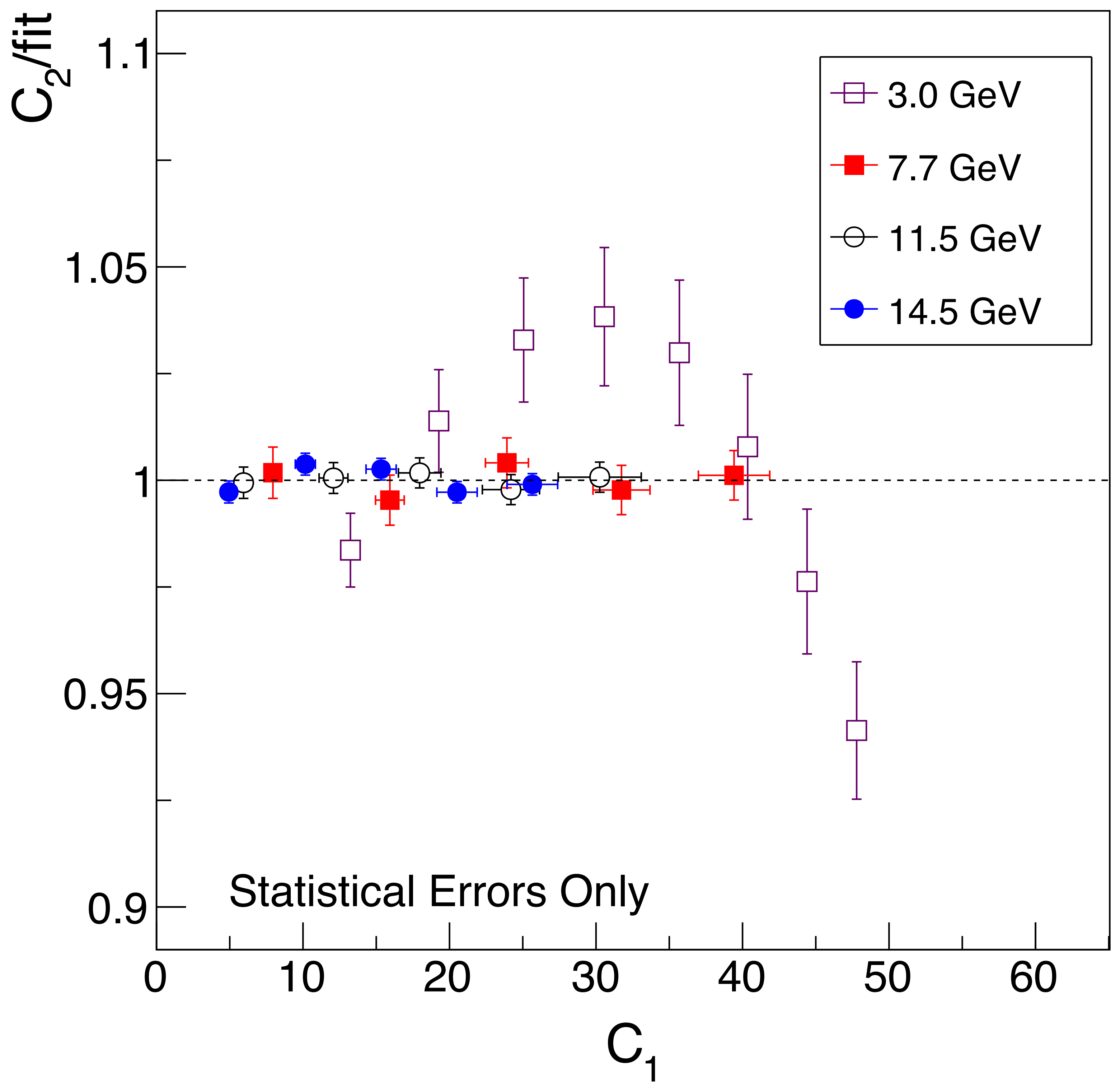
system size = rapidity window W, temperature, chemical potential

The QCD CP from finite-size scaling: Where can we expect scaling?

- For fluids far from the critical region, a mean-field treatment is good enough.
  The transition between the critical scaling region, intermediate scaling region, and extended scaling region has been studied: for fluids, the extended scaling region essentially covers the entire phase diagram where fluctuation contributions are small but finite.
  M.A. Anisimov, S.B. Kiselev, J.V. Sengers, S.Tang, Crossover approach to global critical phenomena in fluids, Physica A 188, 4 (1992)

- In the region of the phase diagram where the bulk of the evolution is well described by a scale free theory (hydrodynamics), the data follows Taylor's Law: $\sigma^2 = a\lambda^p$ (scale free)





$$C_2 = aW^p$$
$$C_2 = a(xW)^p = ax^pW^p = a'W^p$$
where $C_1 \propto W$ in this energy range

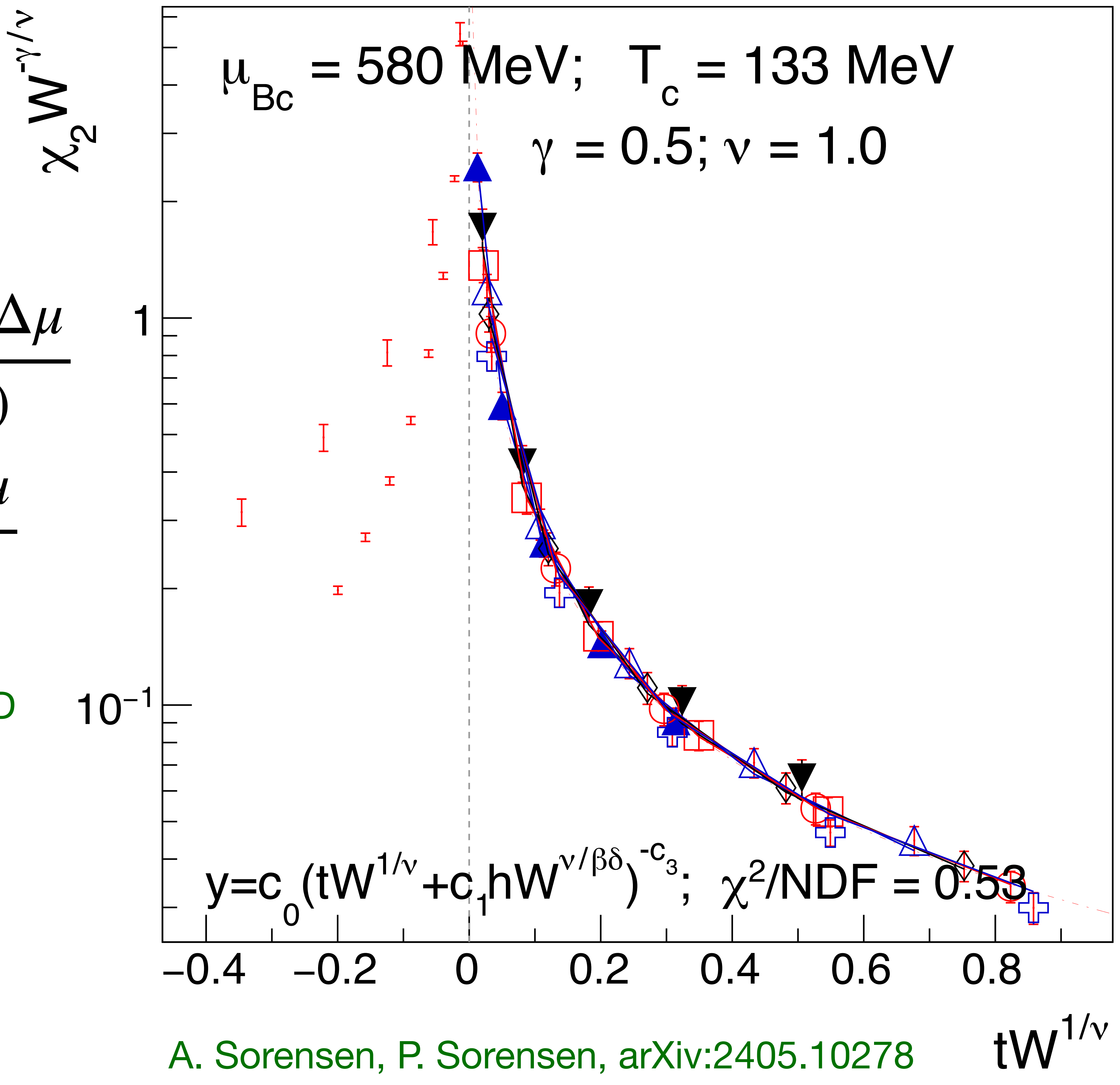
Scale invariance supports the applicability of FSS
(not for collisions at 3 GeV)

The QCD CP from finite-size scaling: 2-D fit w/ mean-field exponents

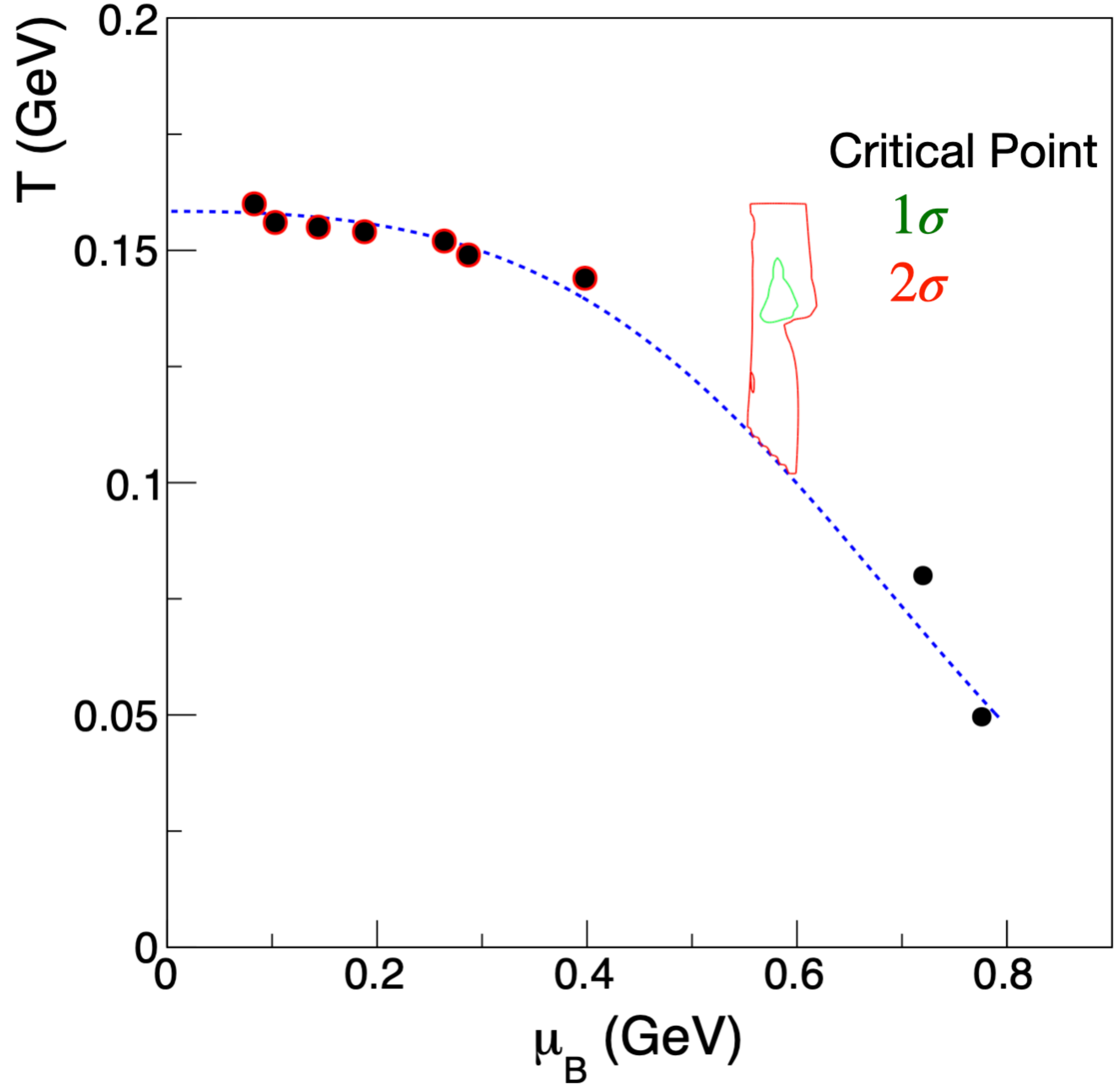
$$h(\mu, T) = -\frac{\cos\alpha_1 \Delta T + \sin\alpha_1 \Delta\mu}{wT_c \sin(\alpha_1 - \alpha_2)}$$

$$t(\mu, T) = \frac{\cos\alpha_2 \Delta T + \sin\alpha_2 \Delta\mu}{\rho w T_c \sin(\alpha_1 - \alpha_2)}$$

M.S. Pradeep, M. Stephanov, Phys. Rev. D **100** 5, 056003 (2019) arXiv:1905.13247



A. Sorensen, P. Sorensen, arXiv:2405.10278



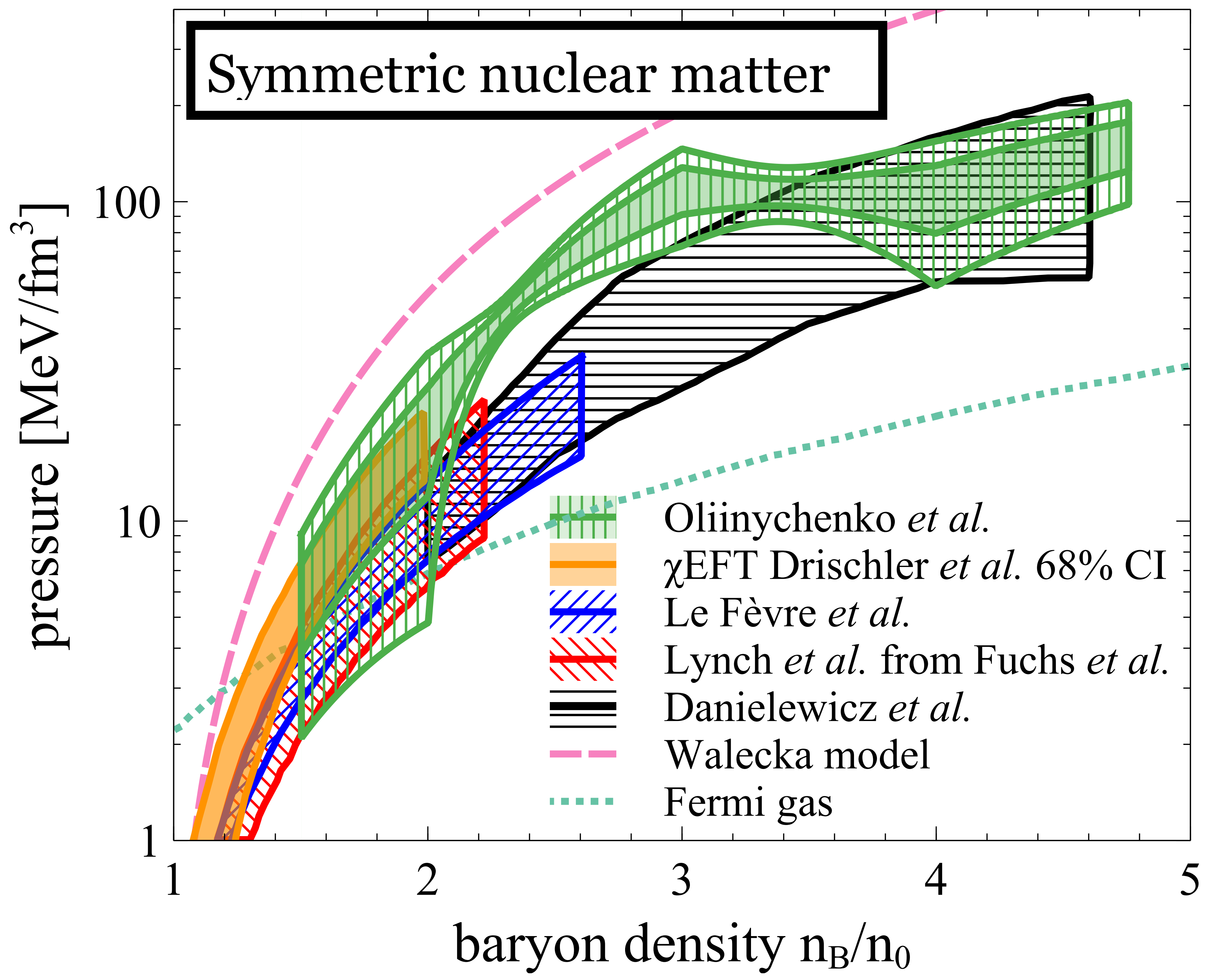
Using mean field exponents, we find scaling for $555 < \mu_{B,c} < 610$ MeV; T_c only constrained by "plausibility" (below $T_{pc}(\mu_B = 0)$ and above $T_{\rm fo}$)

Chi-square contours identify an allowed region in the phase diagram

$\mu_{B,c} = 580 \pm 30$ MeV

Summary

- New data from BES-II is here and/or imminent: see the next talk by Xin Dong
- Multiple studies point to QCD CP in region of the phase diagram probed by BES FXT ($\sqrt{s_{NN}} \approx 4.5$ GeV)



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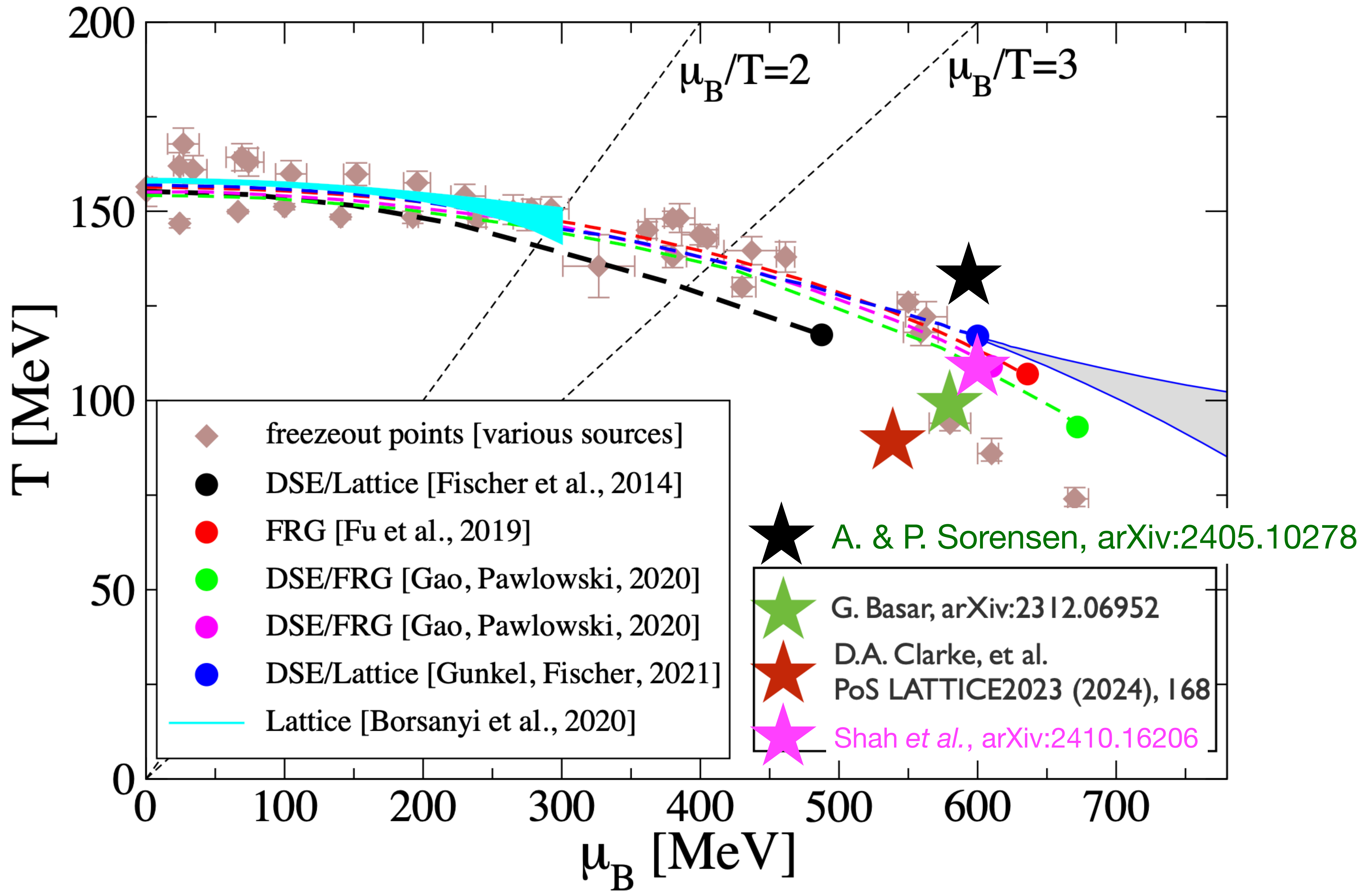

Thank you for your attention