Constraining the EOS from flow and the QCD CP from proton fluctuations

Agnieszka Sorensen

Facility for Rare Isotope Beams

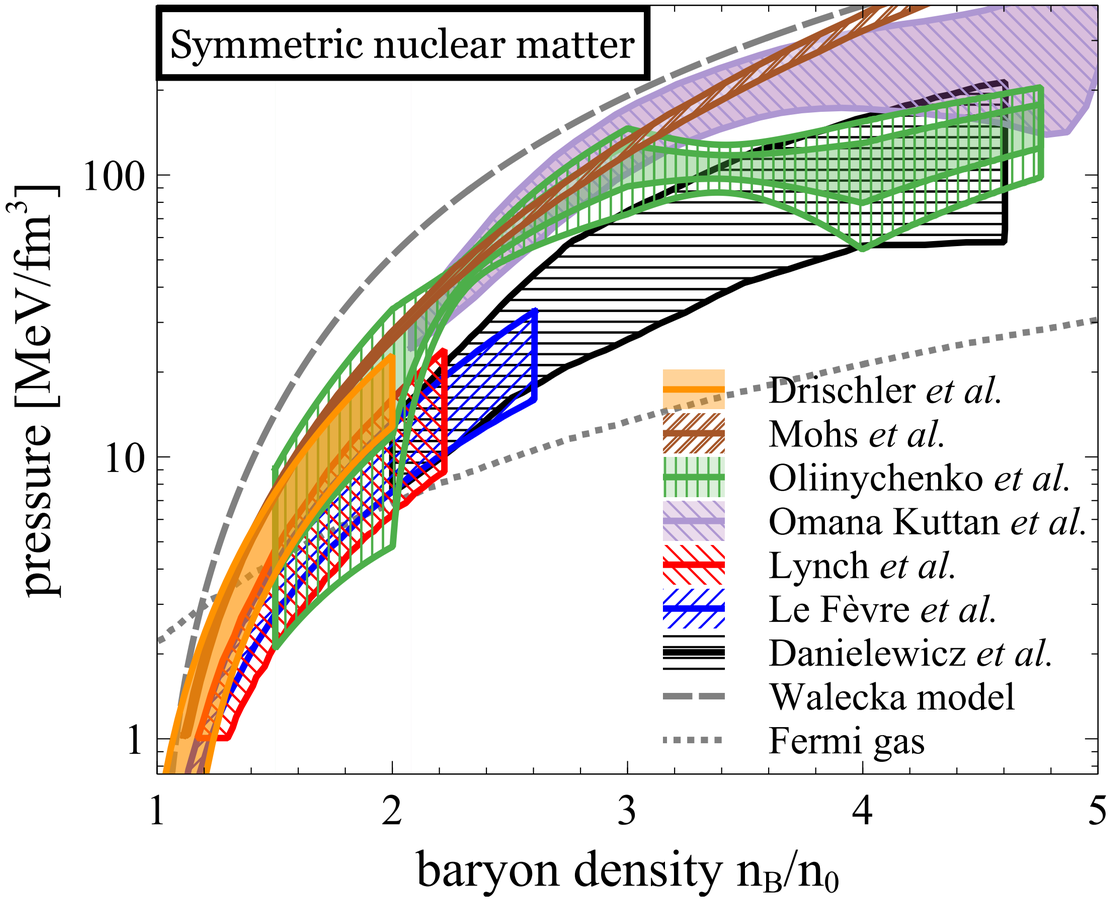
Michigan State University

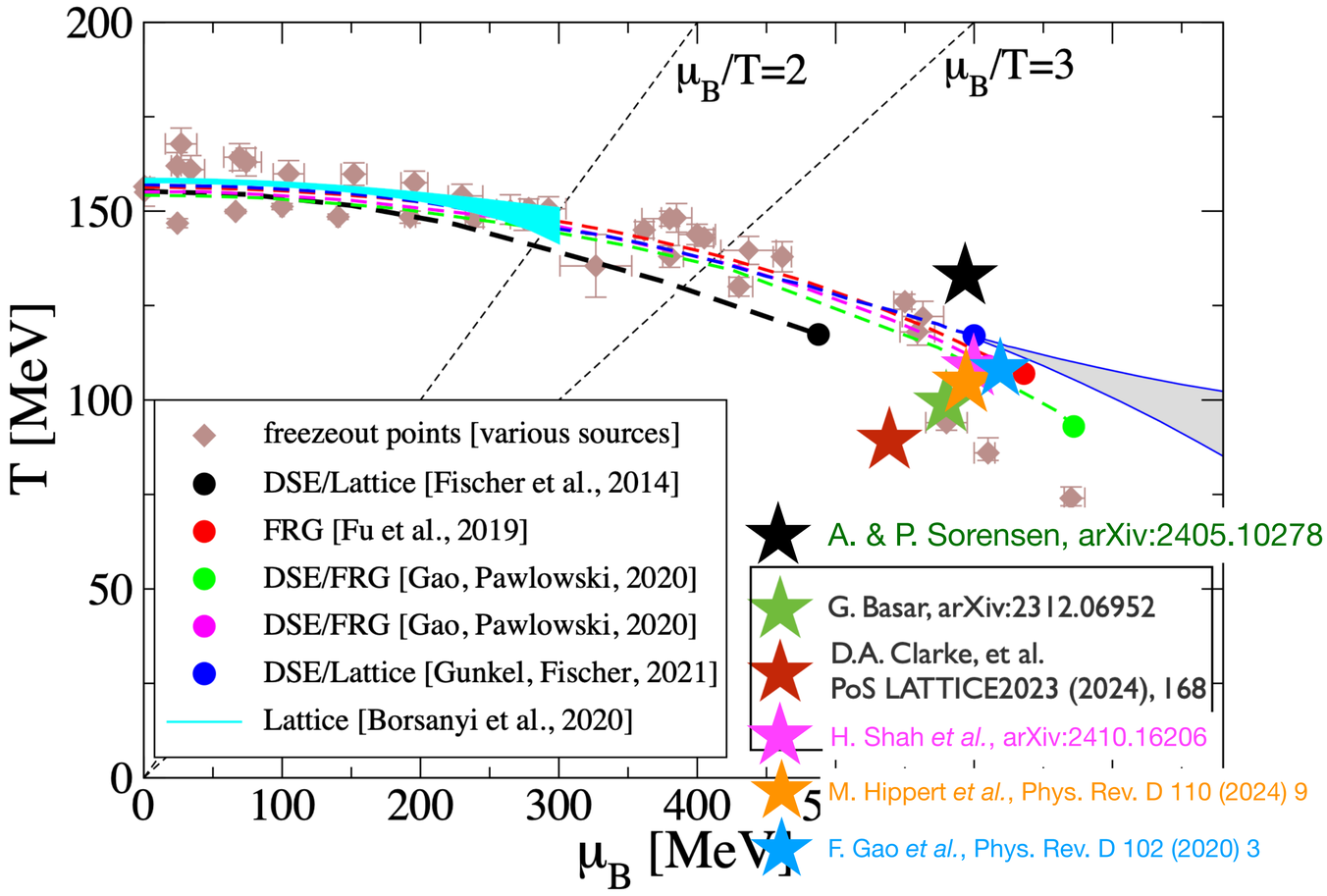
Phys. Rev. C 108 (2023) 3, 034908 (arXiv:2208.11996)
arXiv:2405.10278

FRIB

Selected recent results

- constraints on the n_B-dependence of the EOS: χEFT, transport model comparisons: old (3) and new (3)
- constraints on the QCD critical point: LQCD-based (3), holography+LQCD (1), FRG(1), STAR data (1)





adapted from Christian Fischer, CPOD 2024

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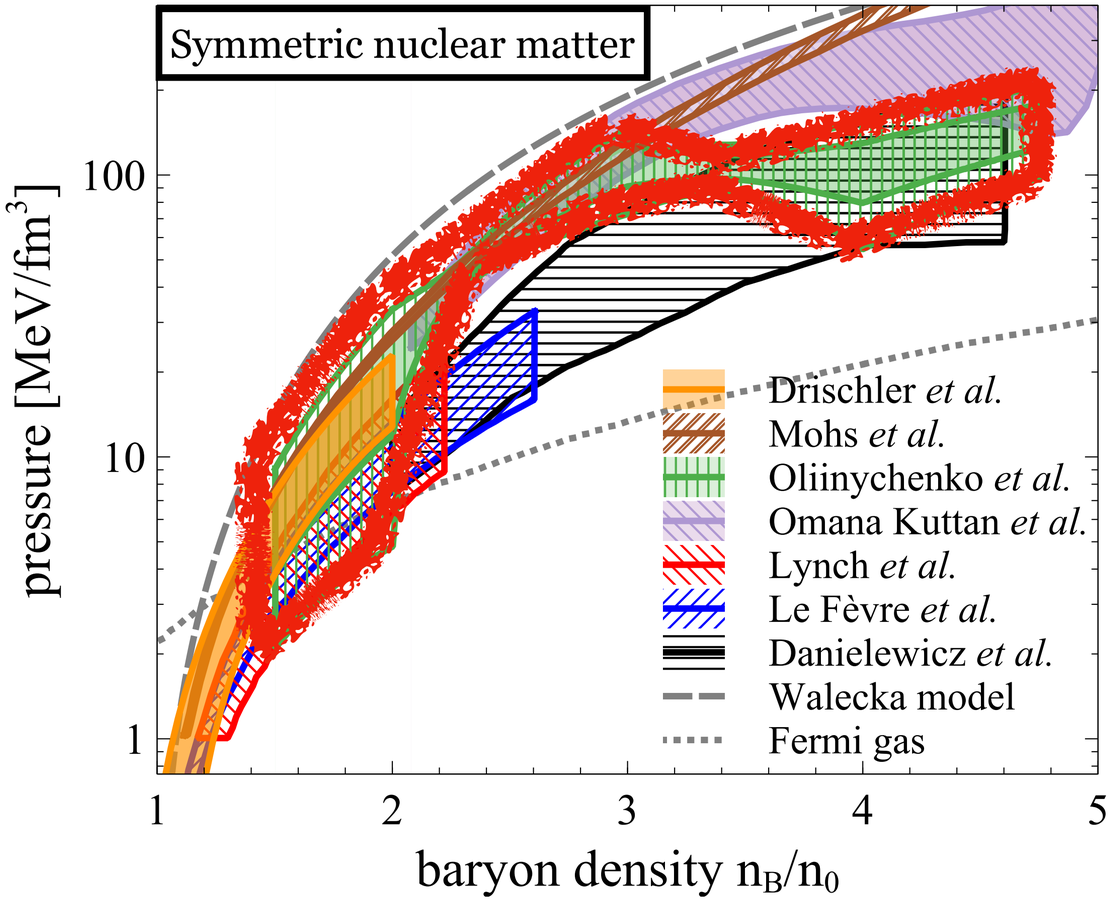
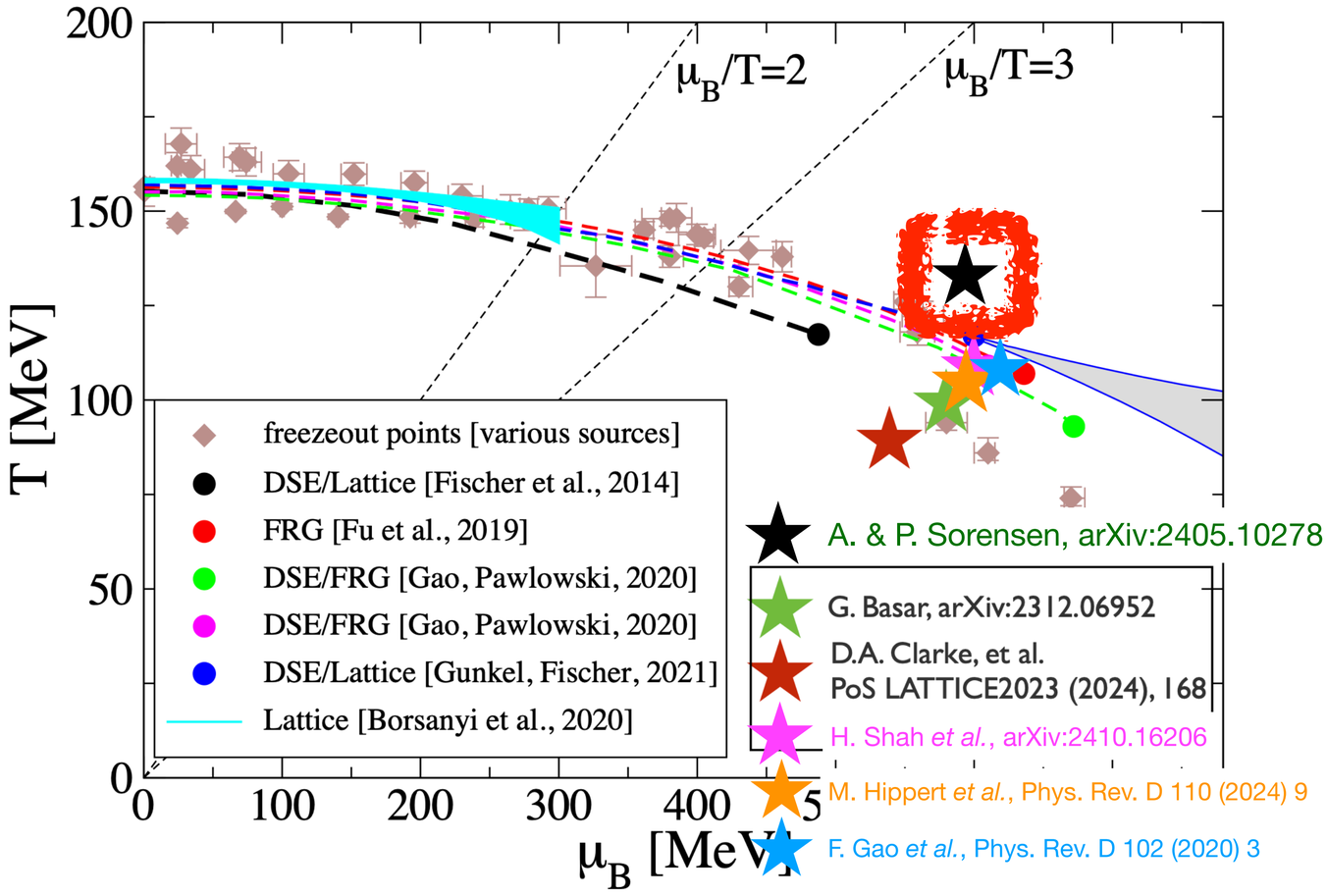
- I'm going to mainly discuss my work





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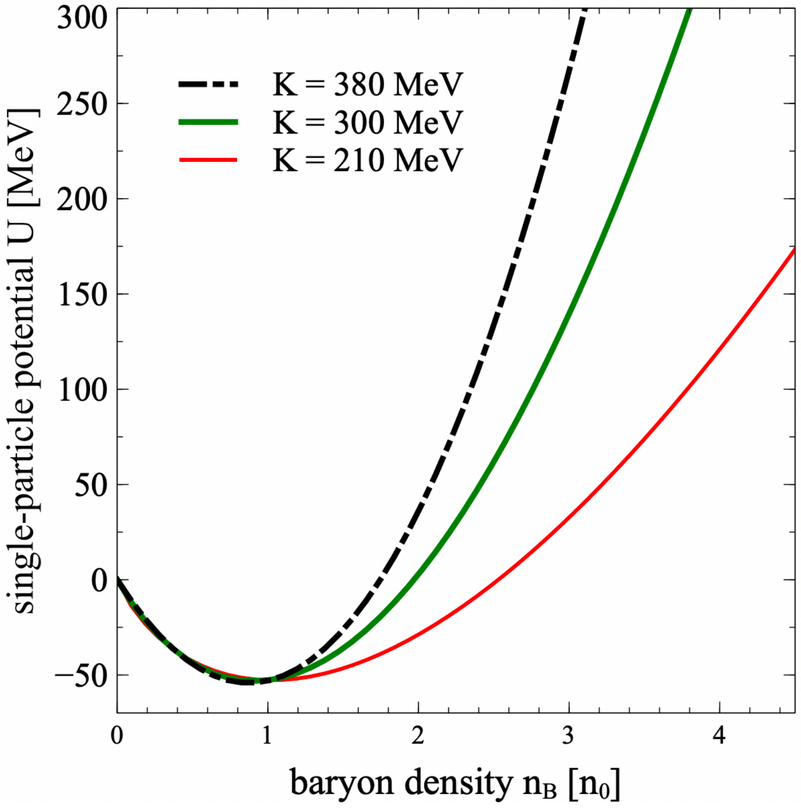
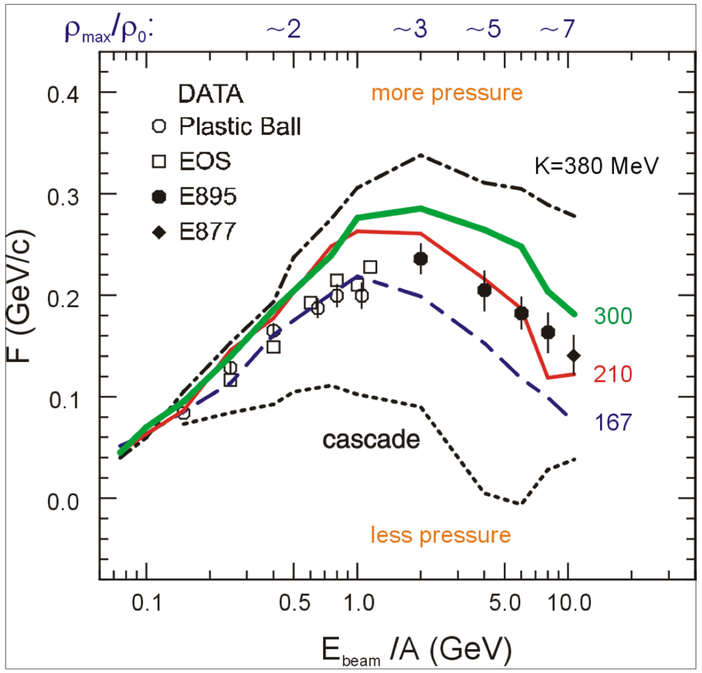
Flow results

Standard way of modeling the EOS: Skyrme potential

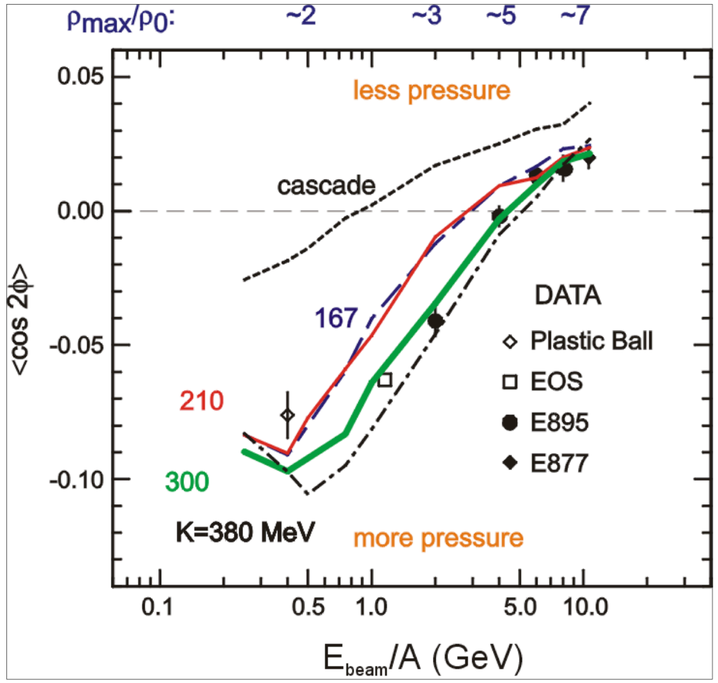
The most common form of the EOS is the "Skyrme potential": $U(n_B) = A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau}$



$$F = \left.\frac{d\langle p_x/A\rangle}{d(y/y_{cm})}\right|_{y/y_{cm}=1} \sim \frac{dv_1}{dy}$$



v_2



P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

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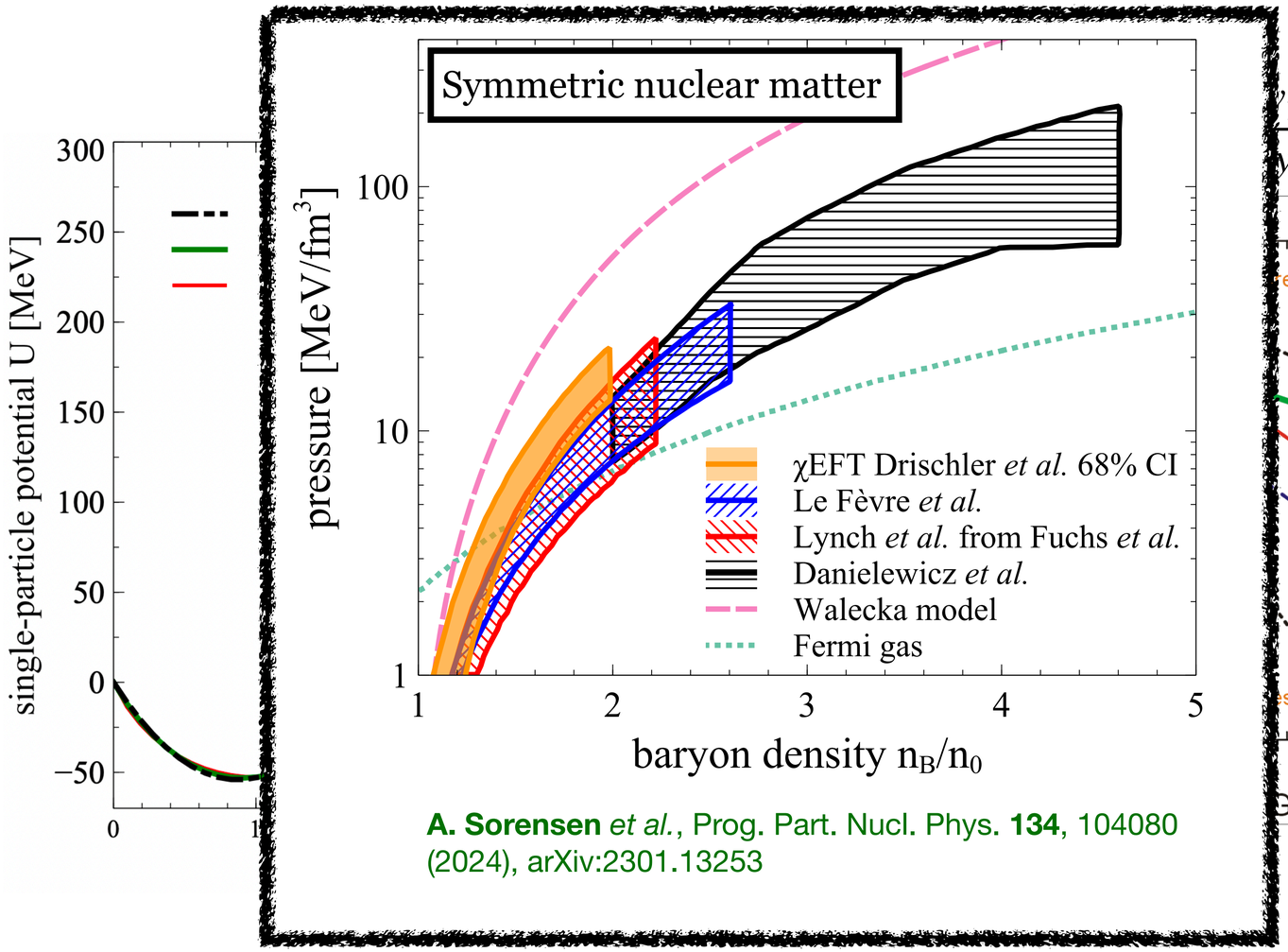
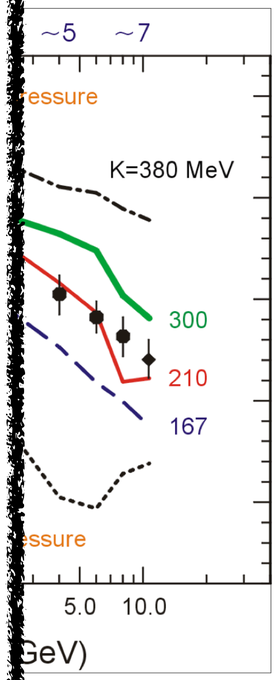
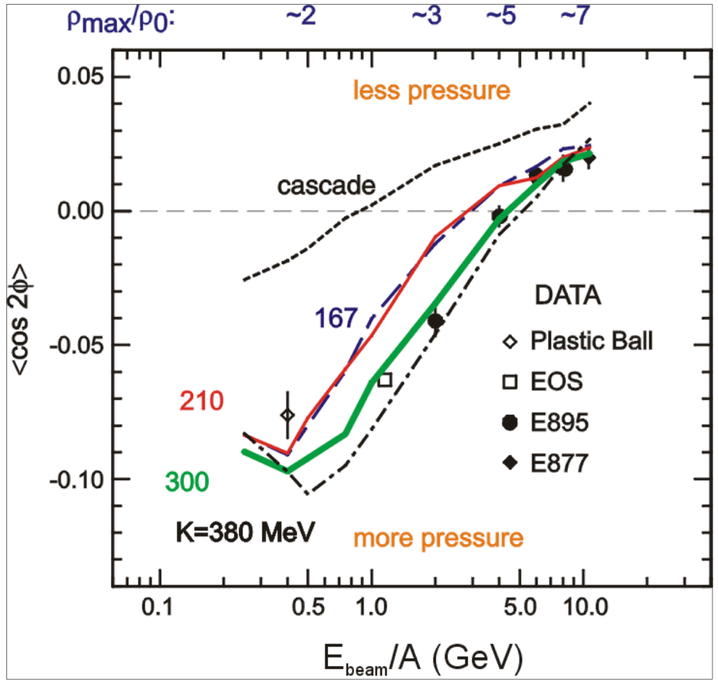


A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024), arXiv:2301.13253





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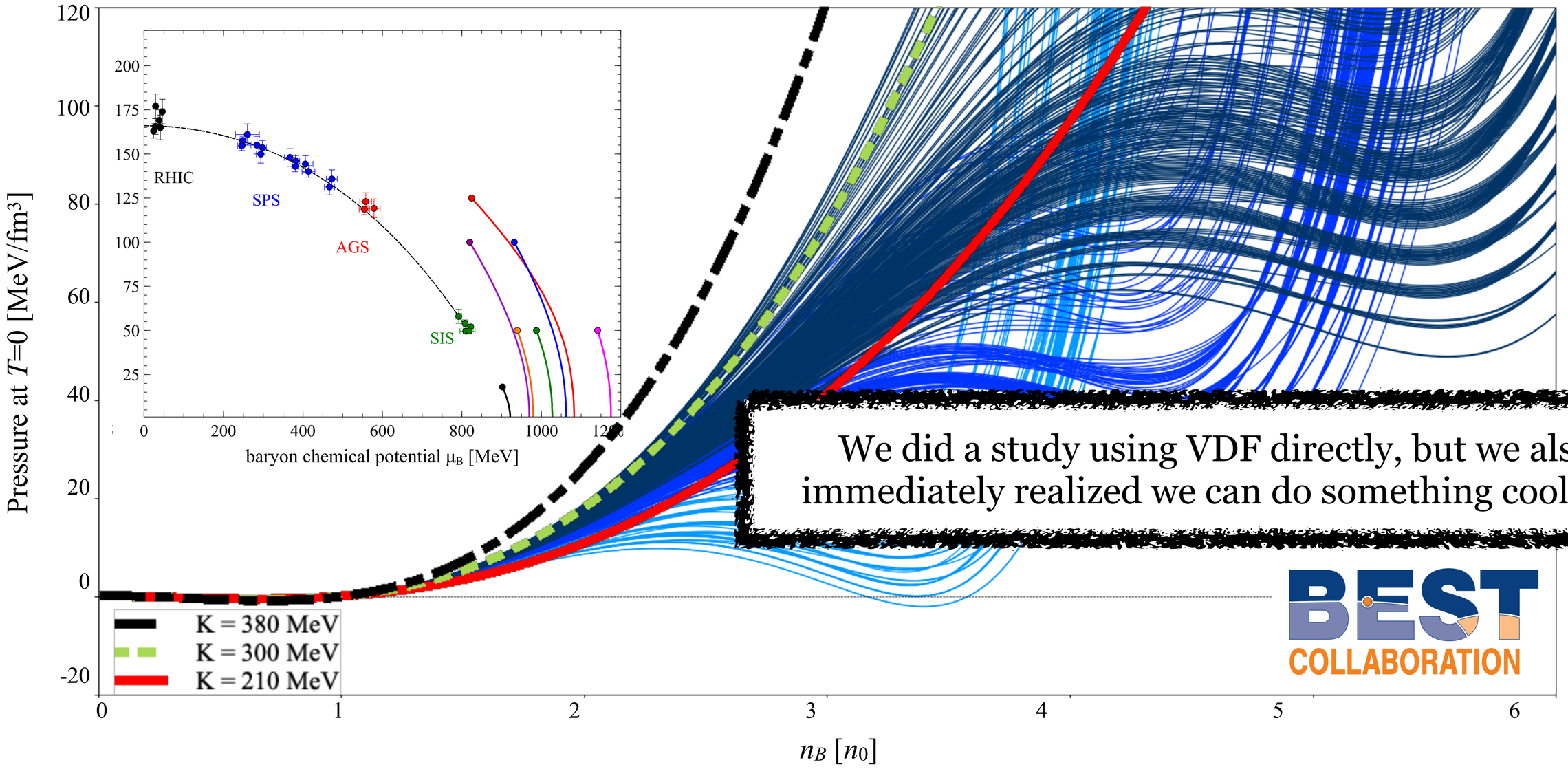
VDF model: relativistic potentials with two 1st order phase transitions

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635



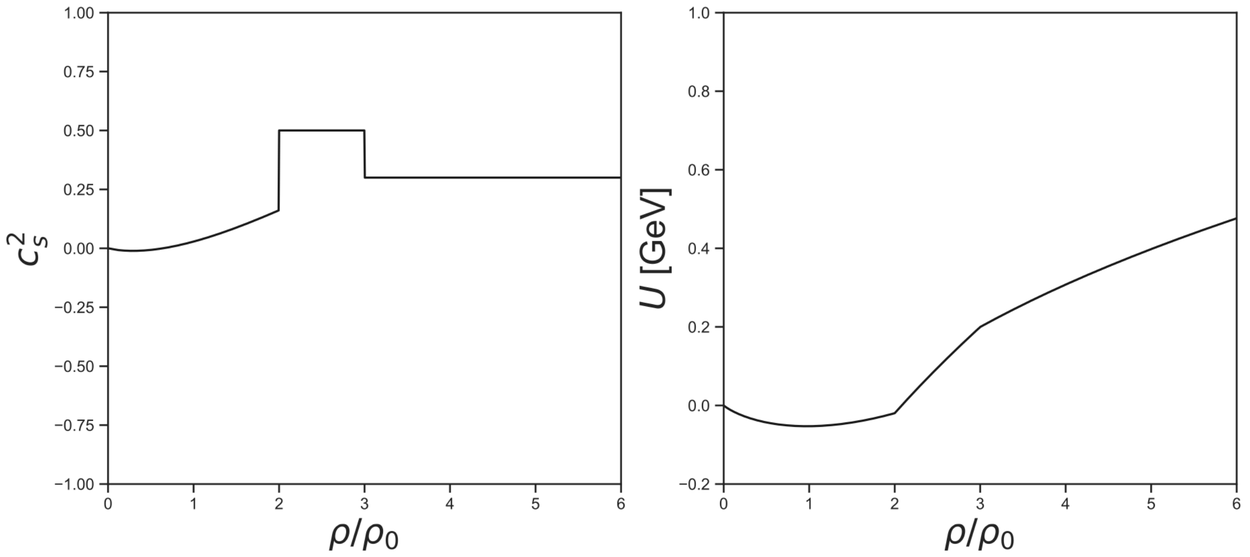
We did a study using VDF directly, but we also immediately realized we can do something cooler...

BEST COLLABORATION

Piecewise parametrization of c_s^2

Piecewise parametrization of $c_s^2(n_B)$:

$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$



1-to-1 relation to the single-particle potential $U(n_B)$:

$$U(n_B) = \begin{cases} U_{\text{Sk}}(n_B) & n_B < n_1 = 2n_0 \\ U_1(n_B) & n_1 < n_B < n_2 \\ \dots & \\ U_k(n_B) & n_k < n_B < n_{k+1} \end{cases}$$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

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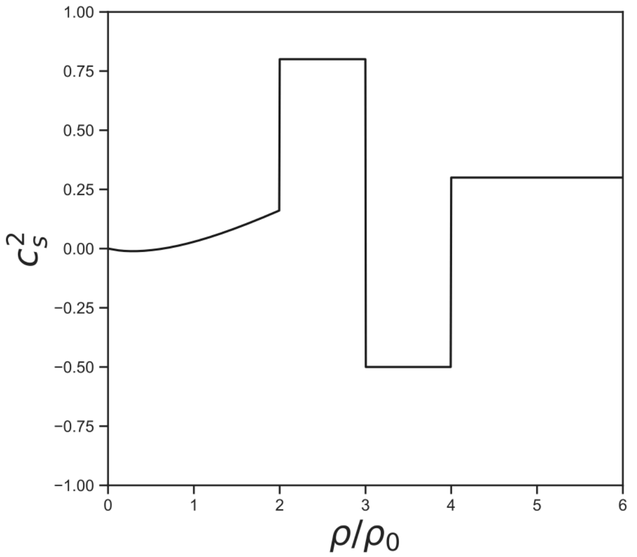

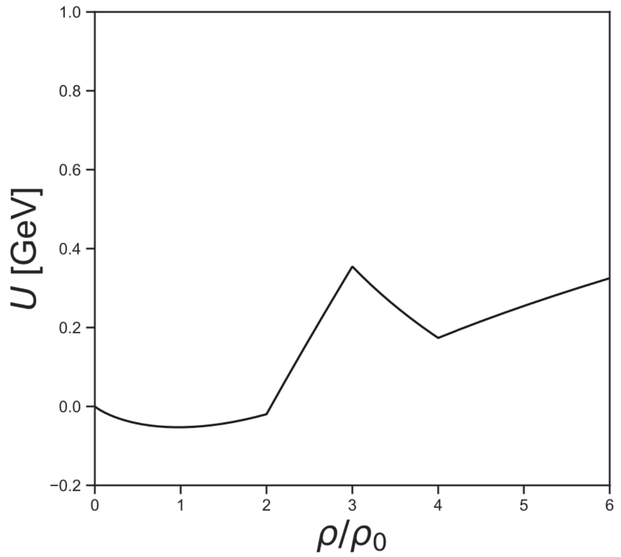

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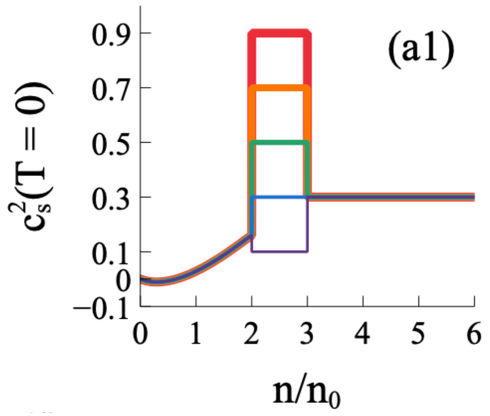

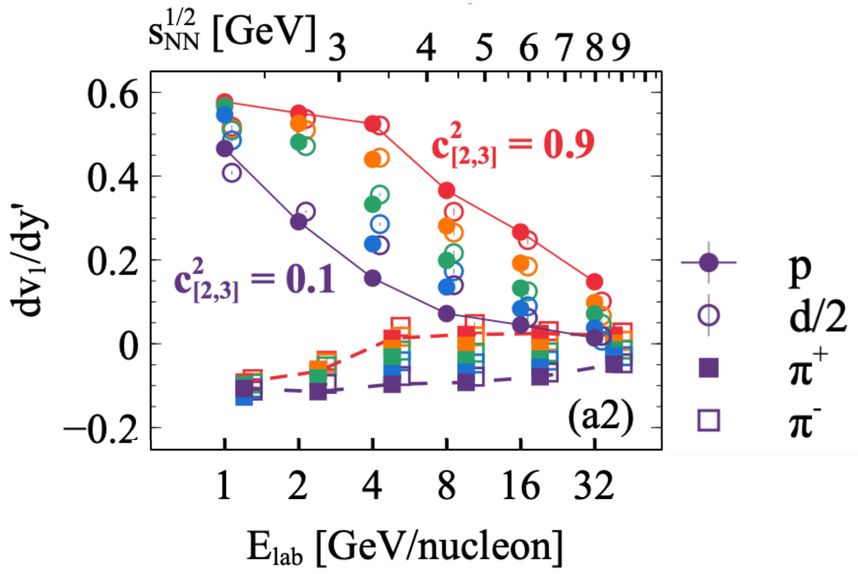

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
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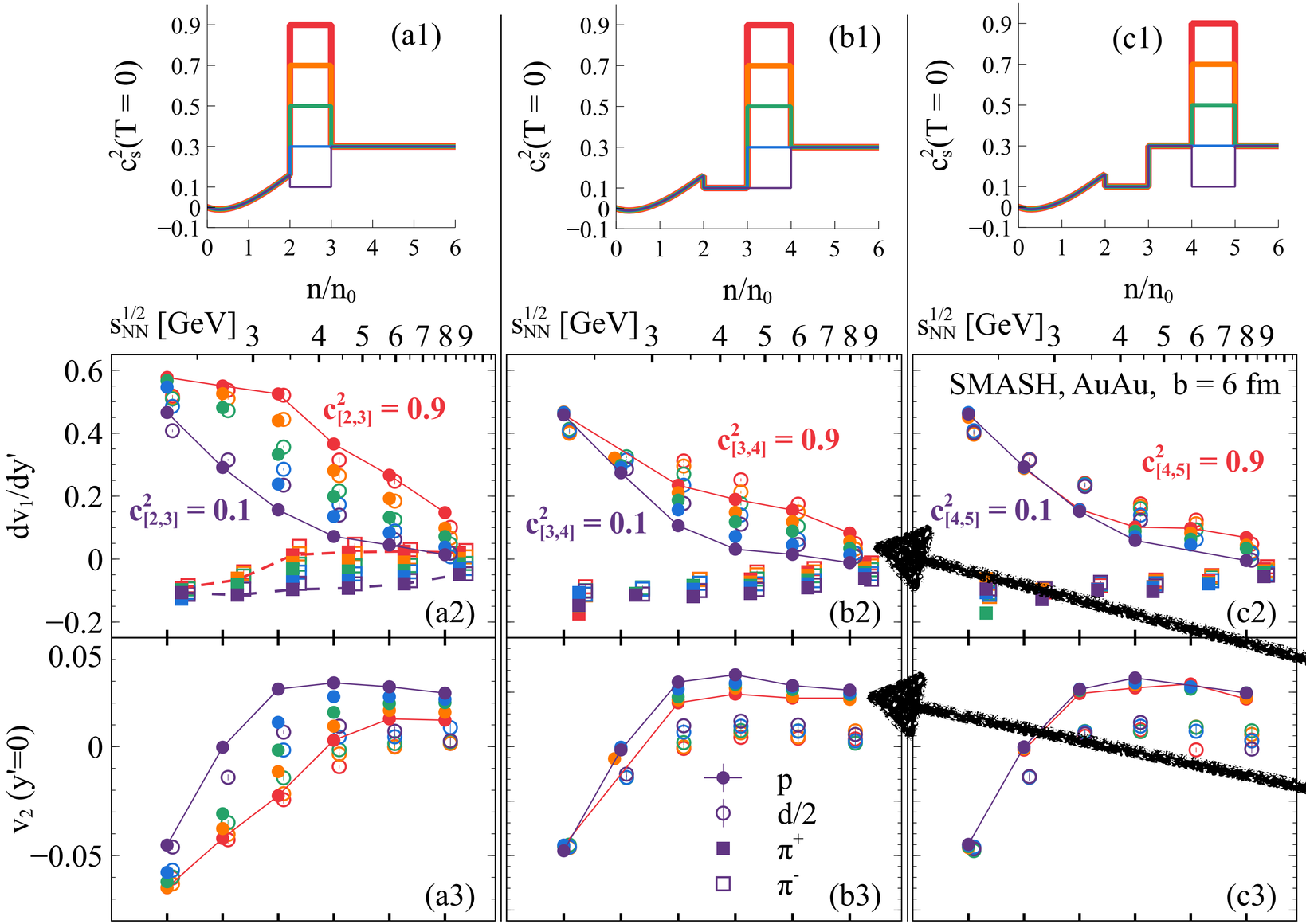
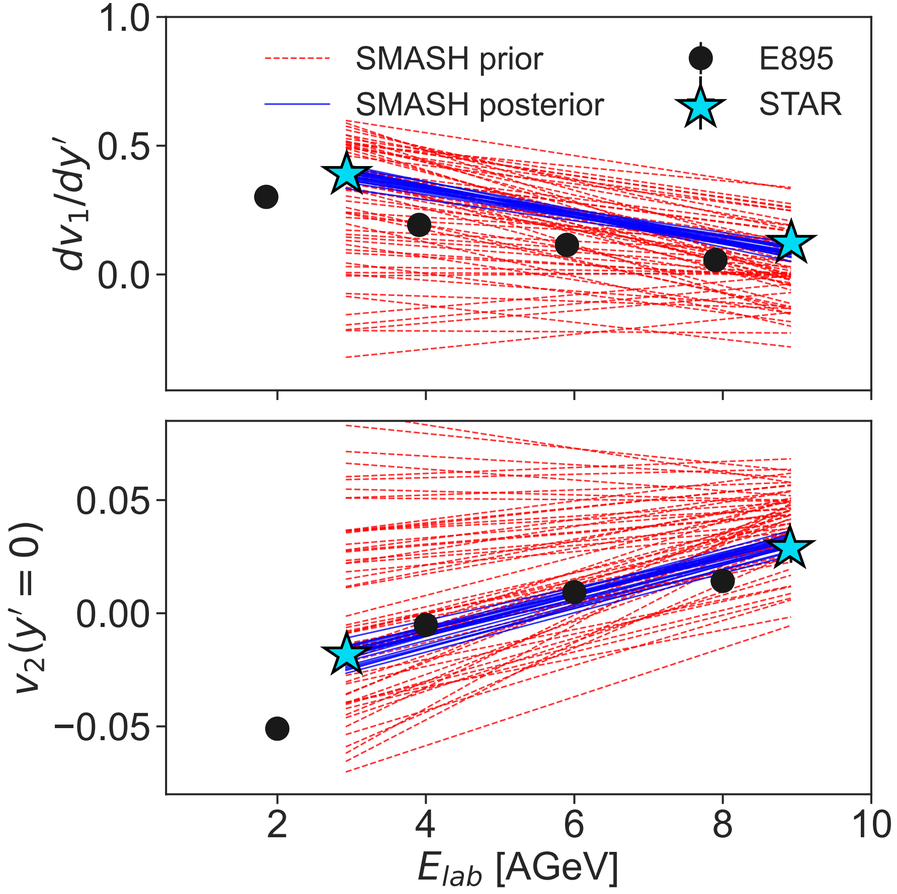
Sensitivity of proton flow to the EOS

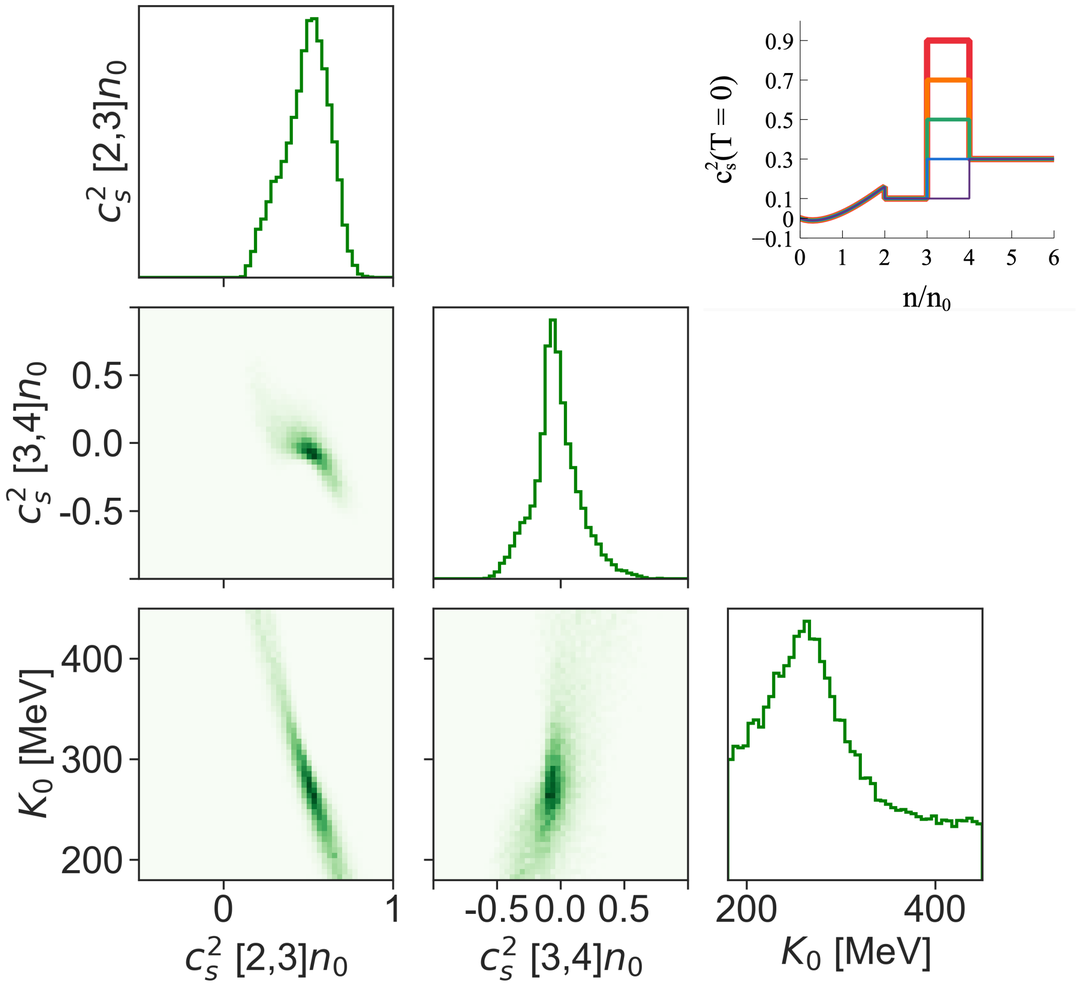
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996



proton v_1 is needed to constrain the EOS at high density

Bayesian analysis of BES flow in SMASH with varying K_0, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$





The maximum a posteriori probability (MAP) parameters are

$K_0 = 285 \pm 67$ MeV, $c^2_{[2,3]n_0} = 0.49 \pm 0.13$, $c^2_{[3,4]n_0} = -0.03 \pm 0.15$

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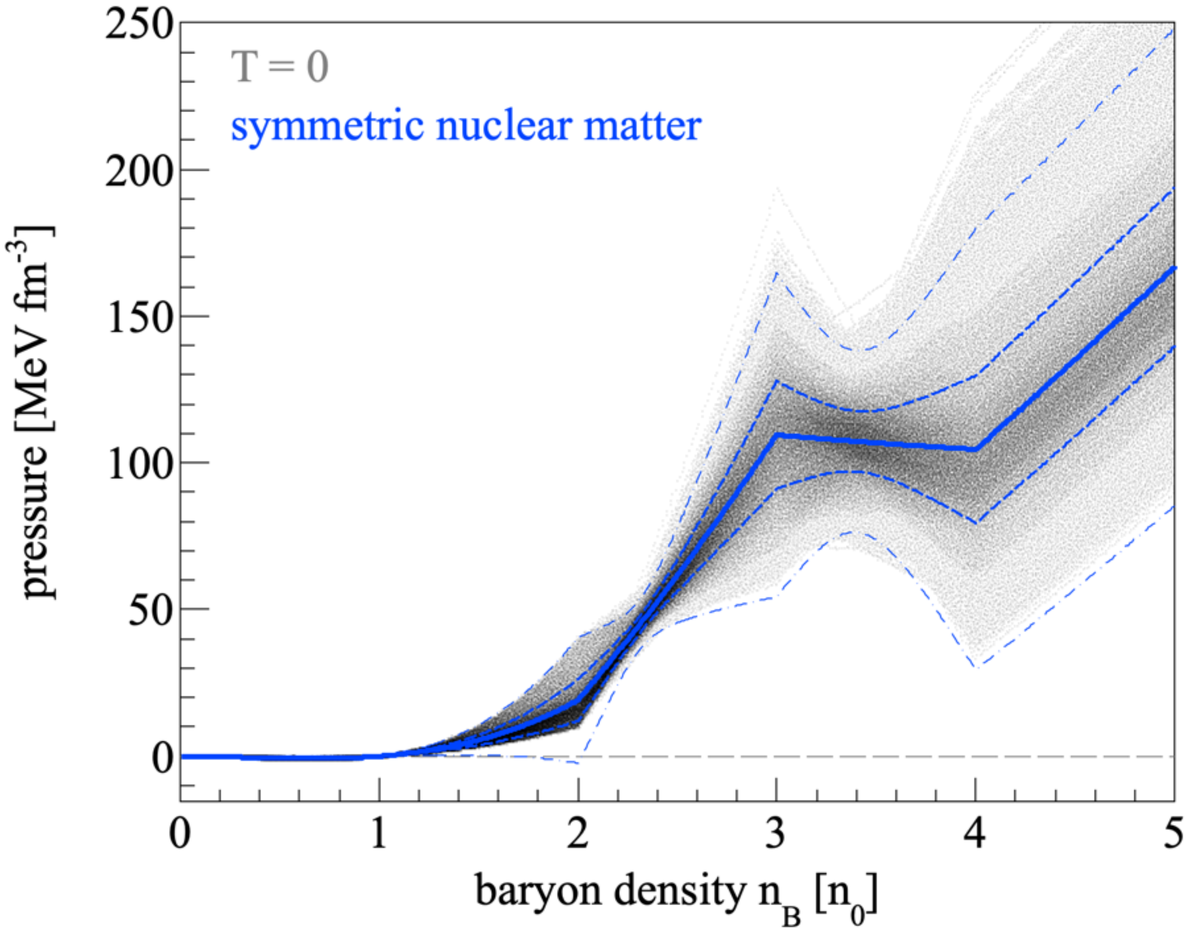
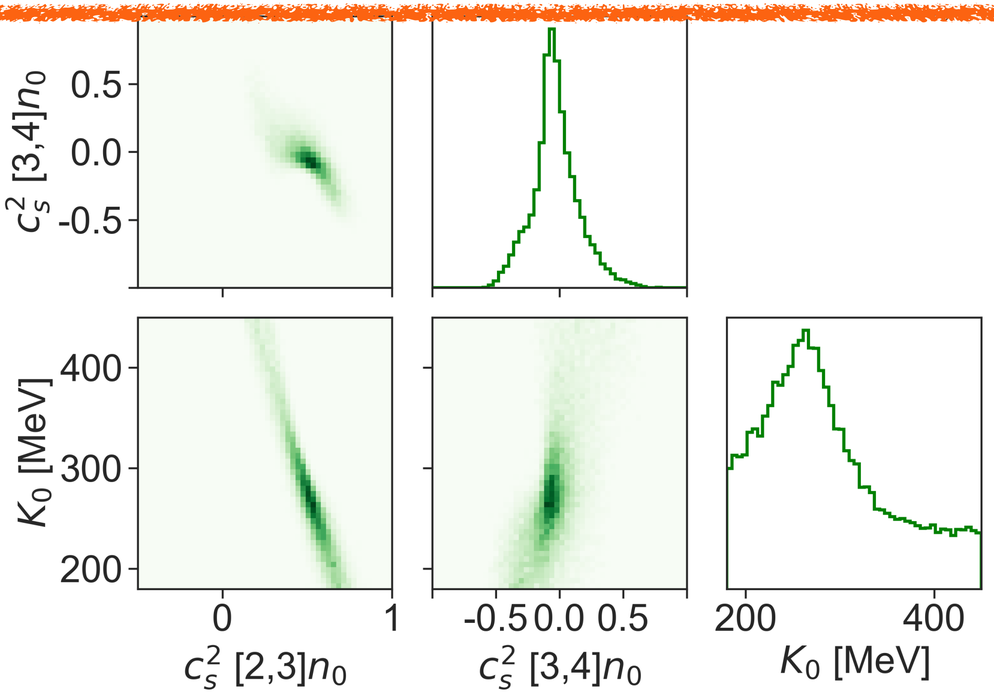


The constrained EOS is very stiff at $n_B \in (2,3)n_0$ and very soft at $n_B \in (3,4)n_0$



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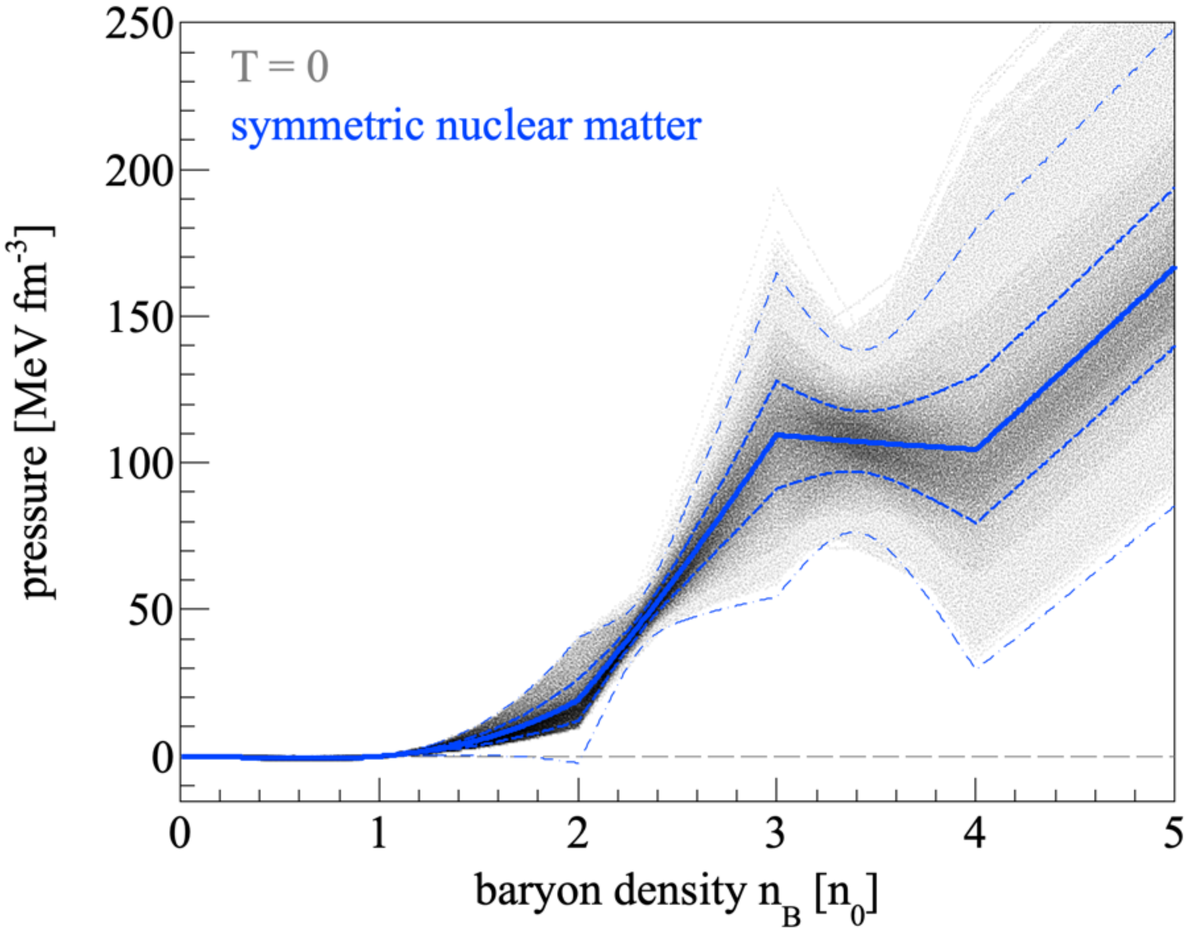

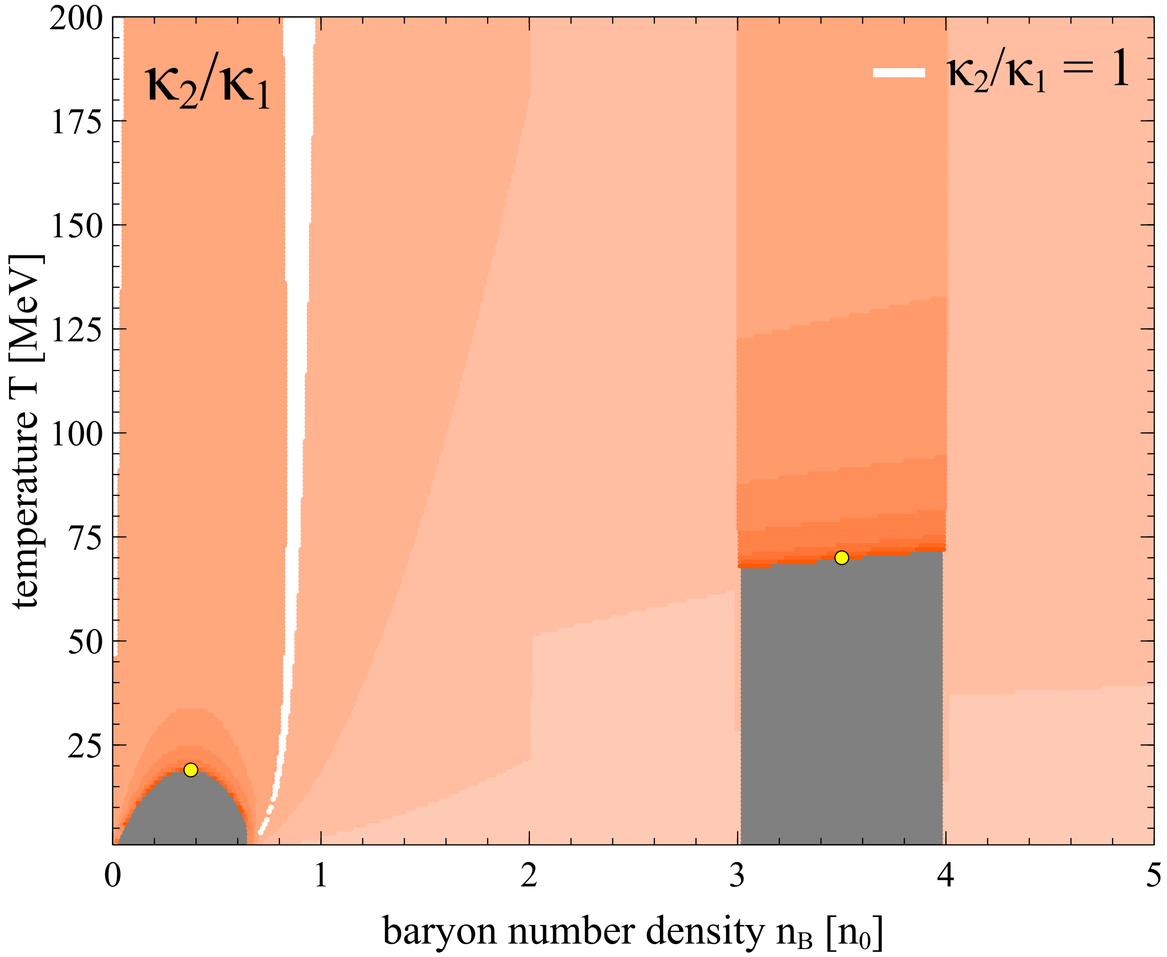

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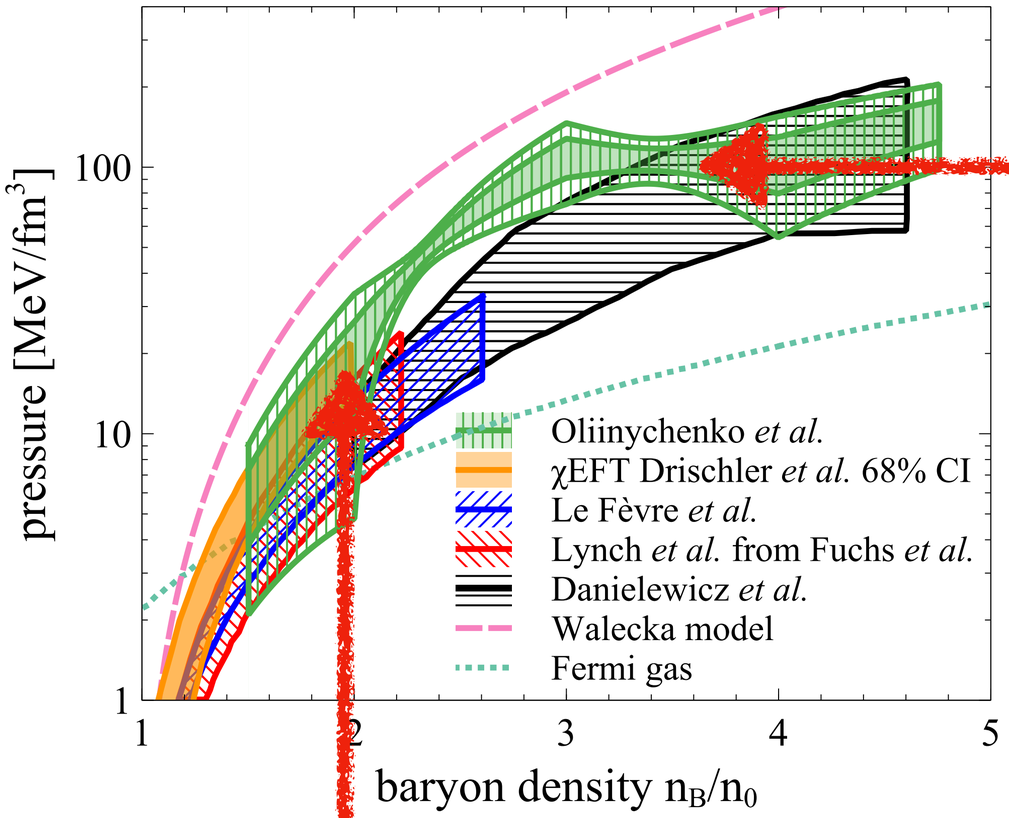
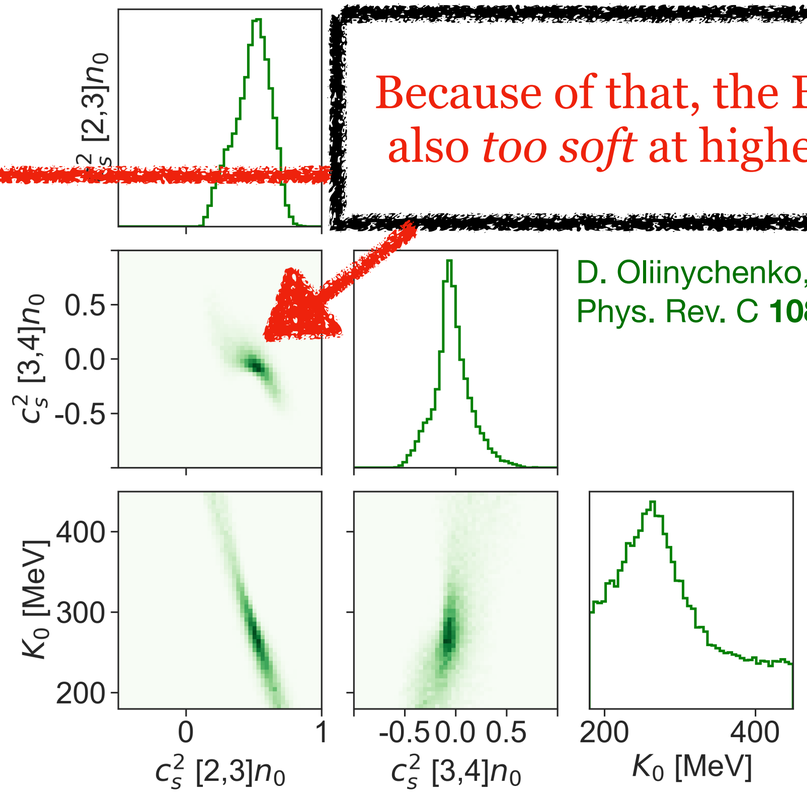
Momentum-dependence of nuclear matter interactions



A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024) arXiv:2301.13253

Without momentum dependence, "artificial" additional source of repulsion is needed = the extracted EOS is too stiff?



Because of that, the EOS may be also *too soft* at higher densities

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, Phys. Rev. C **108**, 3, 034908 (2023), arXiv:2208.11996

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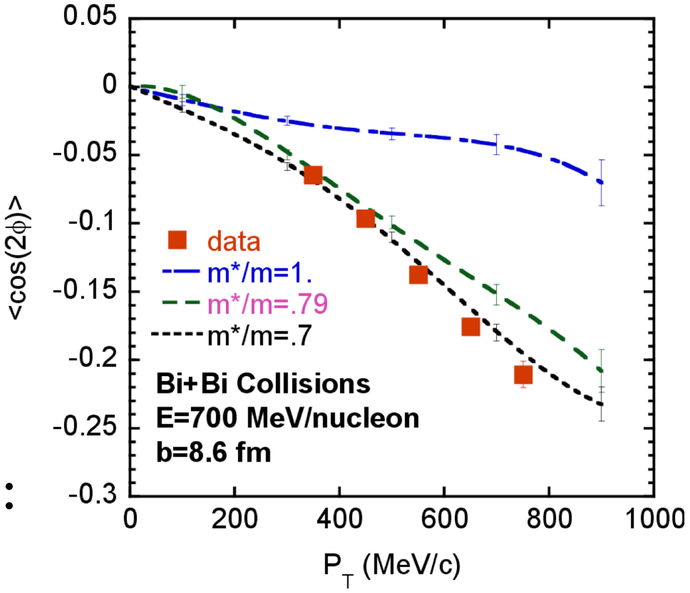
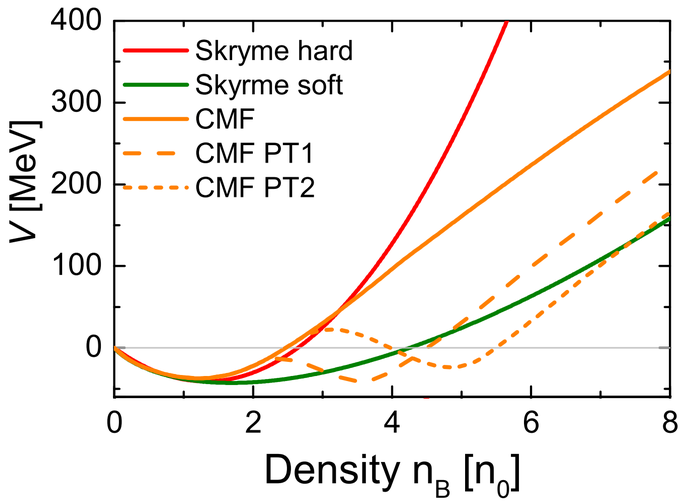
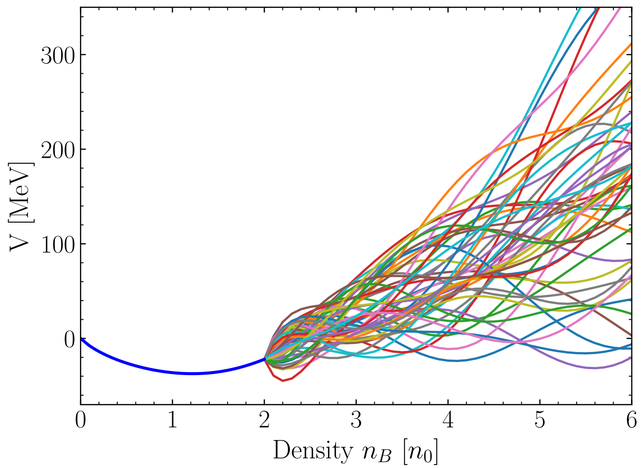


Use peripheral collisions to constrain the *p*-dependence:

Bayesian analysis of flow data in UrQMD





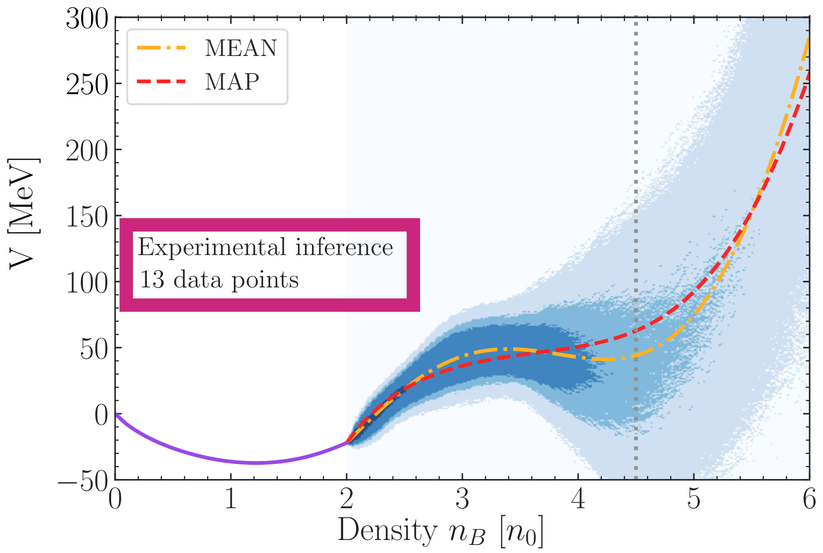
M. Omana Kuttan, J. Steinheimer, K. Zhou, H. Stoecker,
Phys. Rev. Lett. **131** 20, 202303 (2023)
arXiv:2211.11670

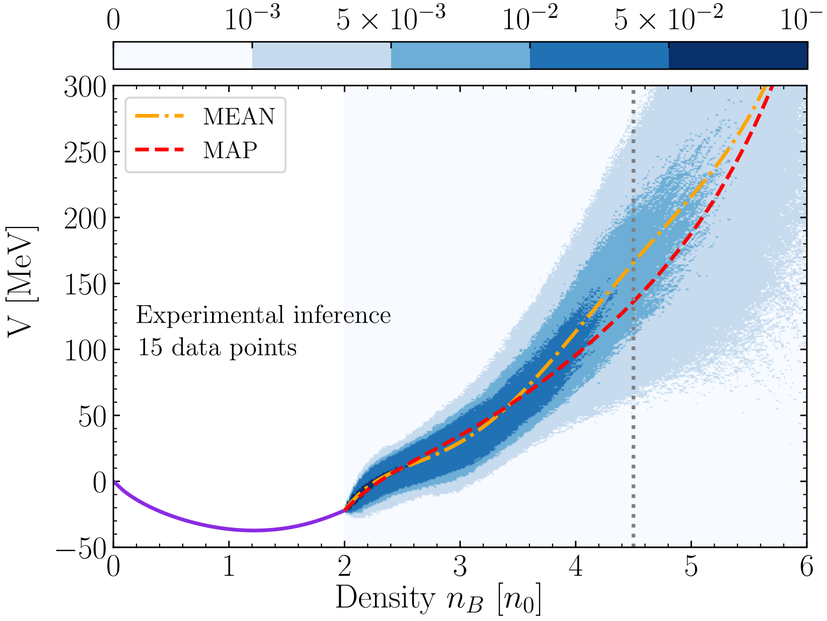
$$V(n_B) = \begin{cases} V_{\mathrm{CMF}} & n_B \leq 2n_0 \\ \sum_{i=1}^{7} \theta_i \left(\frac{n_B}{n_0} - 1 \right)^i + C & n_B > 2n_0 \end{cases}$$

proton mean transverse kinetic energy $\langle m_T \rangle - m_0$:
$\sqrt{s_{\mathrm{NN}}} \in [3.83,\ 8.86]$ GeV

proton elliptic flow v_2 at midrapidity:
$\sqrt{s_{\mathrm{NN}}} \in [2.24,\ 4.72]$ GeV

13 points = excluding $\langle m_T \rangle - m_0$
at the two lowest collision energies
($\sqrt{s_{\mathrm{NN}}} = 3.83,\ 4.29$ GeV)





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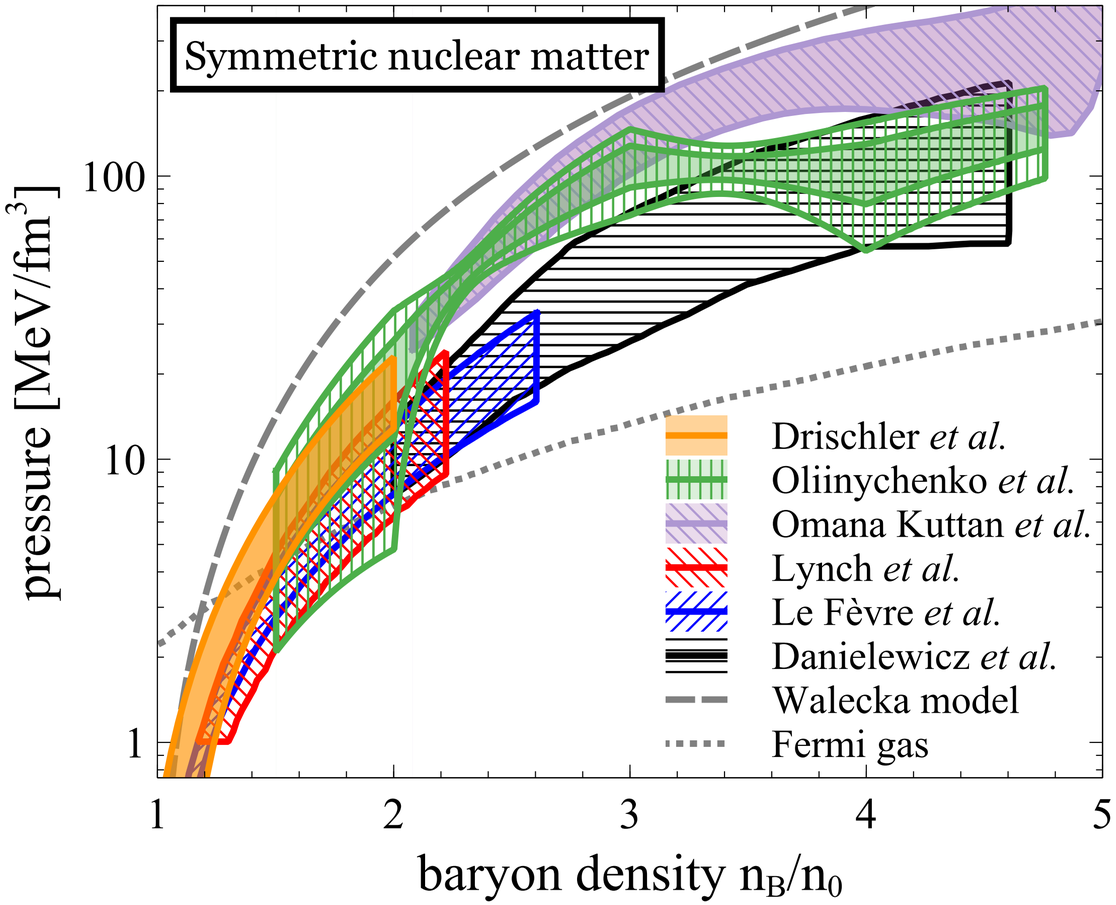

L. Du, **A. Sorensen**, M. Stephanov, Int. J. Mod. Phys. E 33 (2024) 07, 2430008, arXiv: 2402.10183

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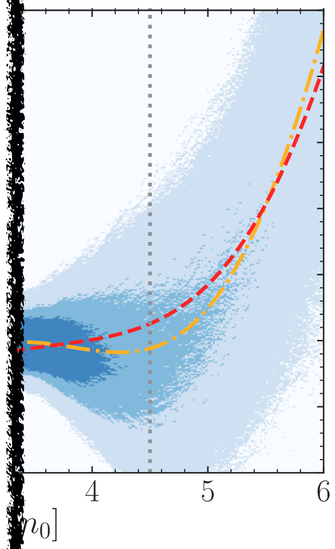
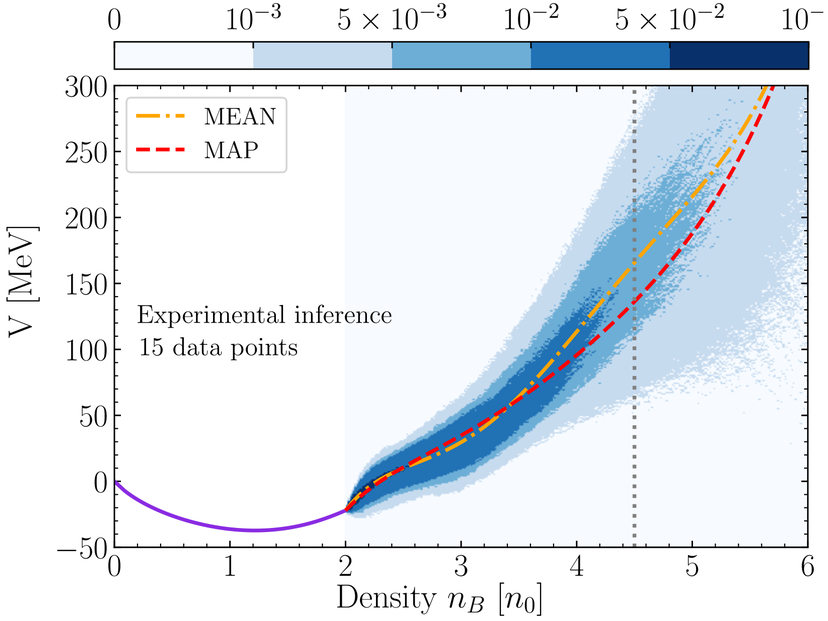
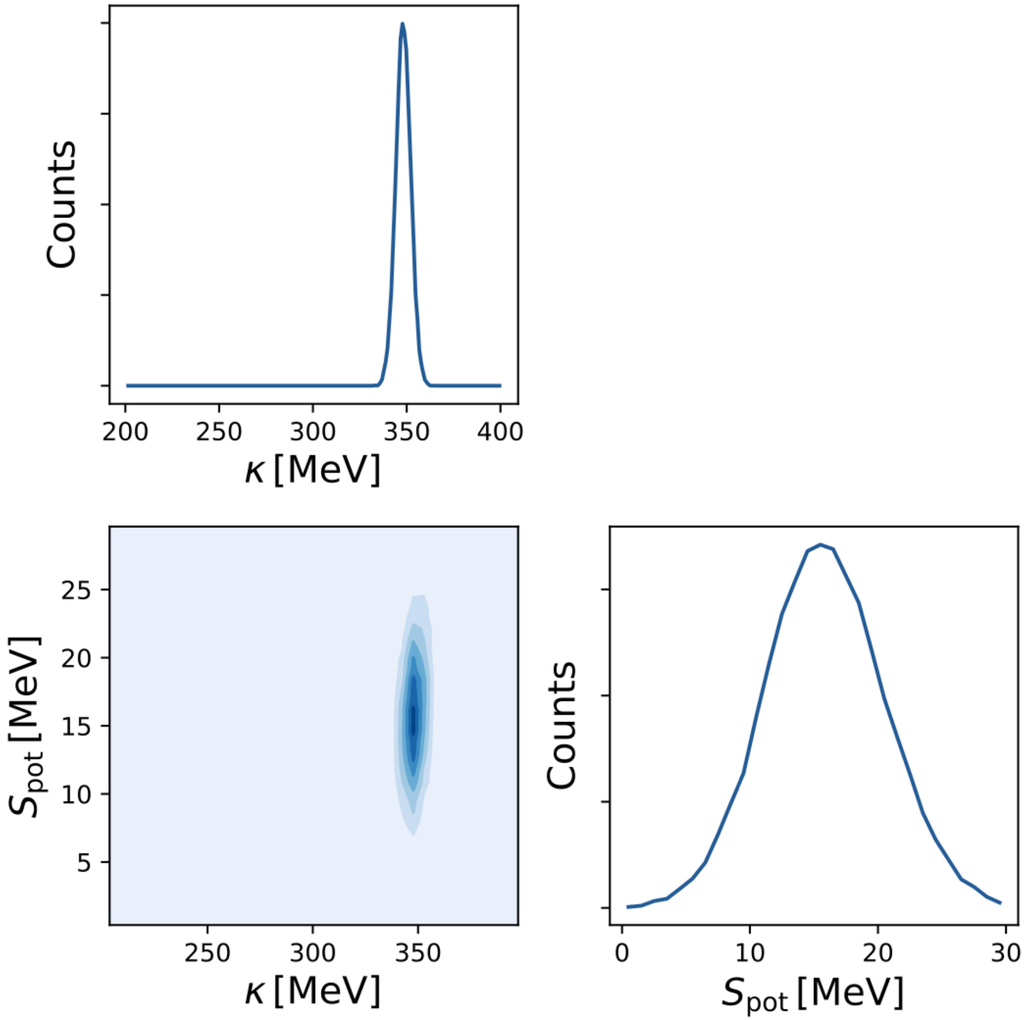


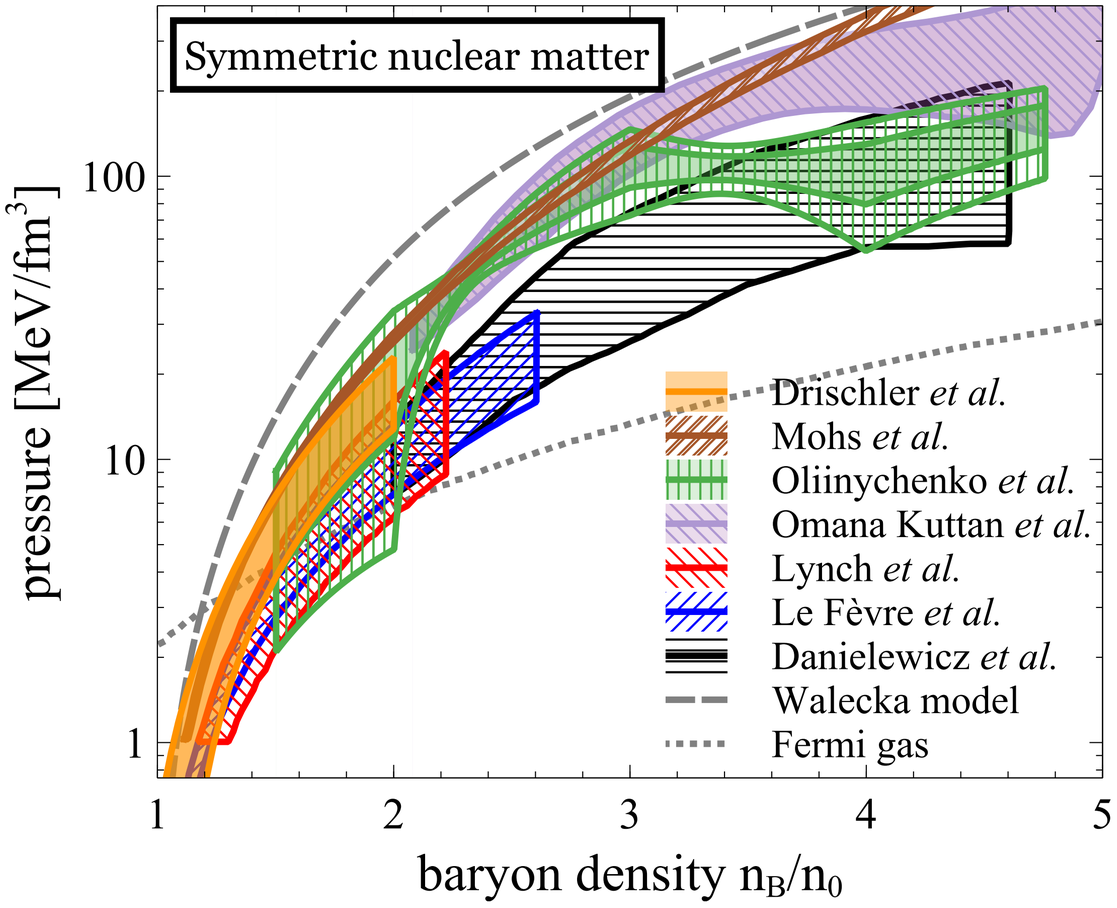


Just in: Bayesian analysis of HADES flow in SMASH

EOS: Skyrme-like with momentum dependence*



J. Mohs, S. Spies, H. Elfner, arXiv:2409.16927



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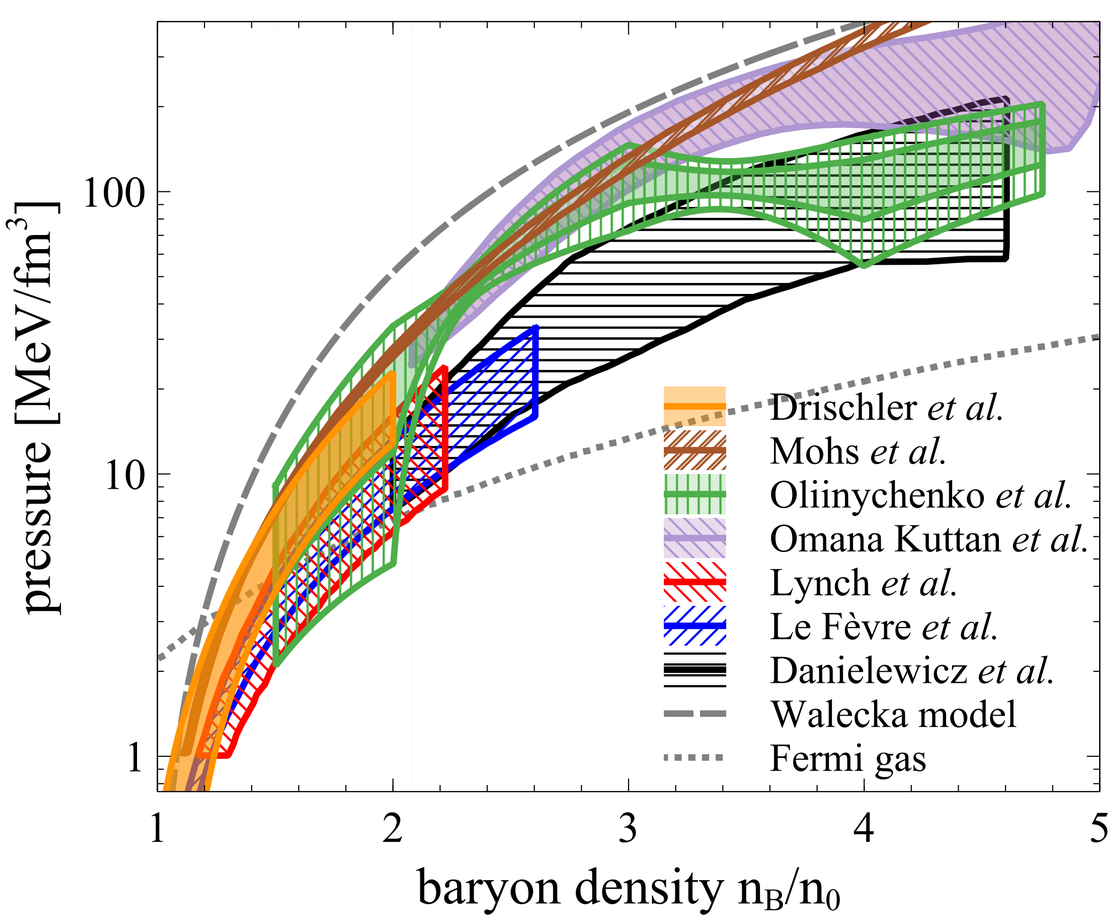
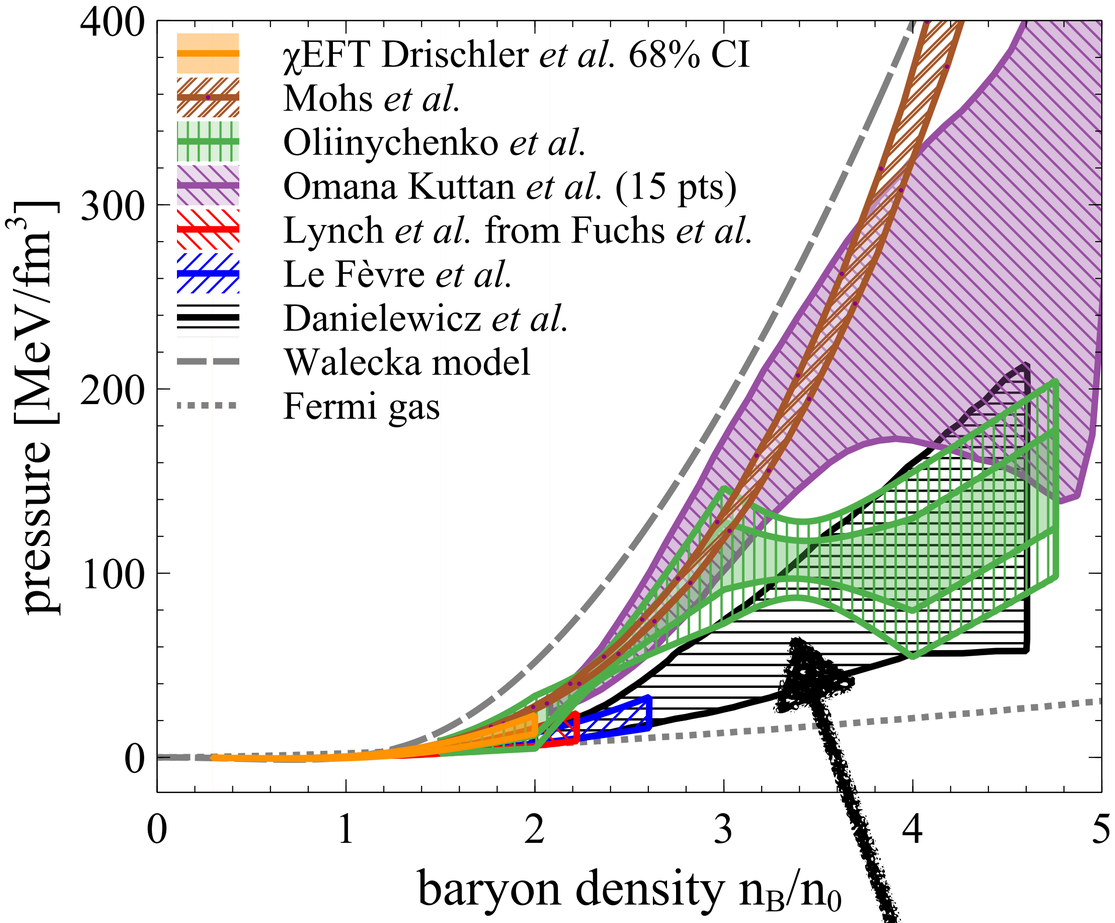
EOS of symmetric nuclear matter: *selected* new results





possible phase transition at $n_B \in (3,4)n_0$

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Finite-size scaling results

Universal behavior

Near CP:

$$c_\infty(t,0) \sim |t|^{-\alpha} \qquad \xi_\infty(t,0) \sim |t|^{-\nu}$$

$$\chi_\infty(t,0) \sim |t|^{-\gamma} \qquad \xi_\infty(0,h) \sim |h|^{-\nu_c}$$

$$t \equiv \frac{T - T_c}{T_c}$$

$$h \equiv \frac{H}{k_B T}$$

For a thermodynamic quantity $X \sim |t|^{-\sigma}$: $X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}}$

Scaling is not unique to critical phenomena.
Scaling law = a physical quantity changes predictably with the scaling of another quantity

E.g., Kepler's third law:
The orbital period of a planet scales as the cube of the semi-major axis of its orbit:
$P^2 = a^3$

The important question for scaling is: what is the scale relevant to the problem?

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CP: infinite volume concept

In real world ξ does not go to infinity = thermodynamic functions do not exhibit singularities

ξ is bound by the size of the system L $\quad \Rightarrow \quad X_L(t_L) \sim L^{\frac{\sigma}{\nu}}$

$$\Rightarrow \quad X_L(t_L) = L^{\frac{\sigma}{\nu}} \phi(t, L) = L^{\frac{\sigma}{\nu}} \phi(tL^{\frac{1}{\nu}})$$

$$\Rightarrow \quad X_L(t_L) L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

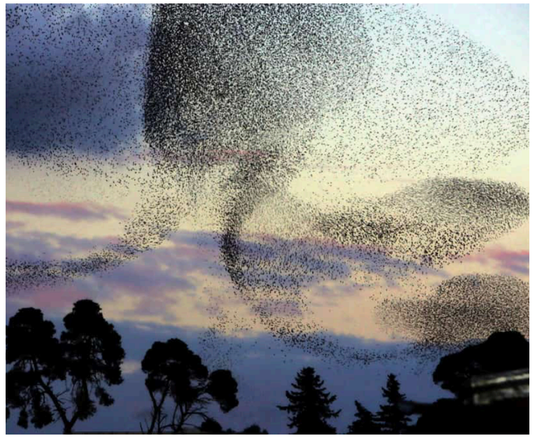
one can find CP by plotting

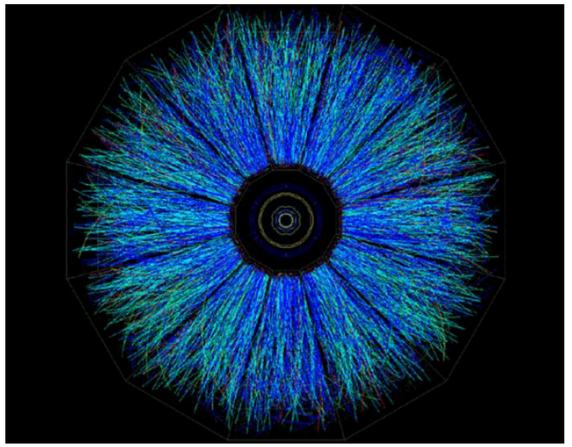
Finite size vs. window size

$$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

Finite-size scaling: change the size of the system, calculate $X_L(t_L)$, repeat

However, changing SIZE is not always possible or doesn't probe the same system:
bird flocks, heavy-ion collisions, …





Solution: study the dependence of X on the size of the *subsystem* that is considered

D. Martin, T. Ribeiro, S. Cannas, *et al.*, Box scaling as a proxy of finite size correlations, Sci Rep 11, 15937 (2021)

THIS WORK: the size of the *subsystem* = rapidity bin width W

A. Sorensen, P. Sorensen, arXiv:2405.10278

Thermal model

- We consider finite-size scaling of proton number susceptibility:

$$\chi_2 = \frac{C_2}{T^3 V} \quad \Rightarrow \quad \chi_2(W, \mu_{\mathrm{fo}}) = \frac{C_2(W, \mu_{B,\mathrm{fo}})}{T_{\mathrm{fo}}^3 W dV_{\mathrm{fo}}/dy}$$

- We use rapidity bin width W as the subsystem size
- We use published thermal model fits for T_{fo} and $\mu_{B,\mathrm{fo}}$
- We parameterize dV_{fo}/dy from several publications
  (for 2.4 GeV, $T_{\mathrm{fo}}^3 V$ is highly uncertain, ranging from about 65 to 650)
- Experiments can improve results by publishing dV_{fo}/dy, T_{fo}, $\mu_{B,\mathrm{fo}}$ from thermal model fits for specific W

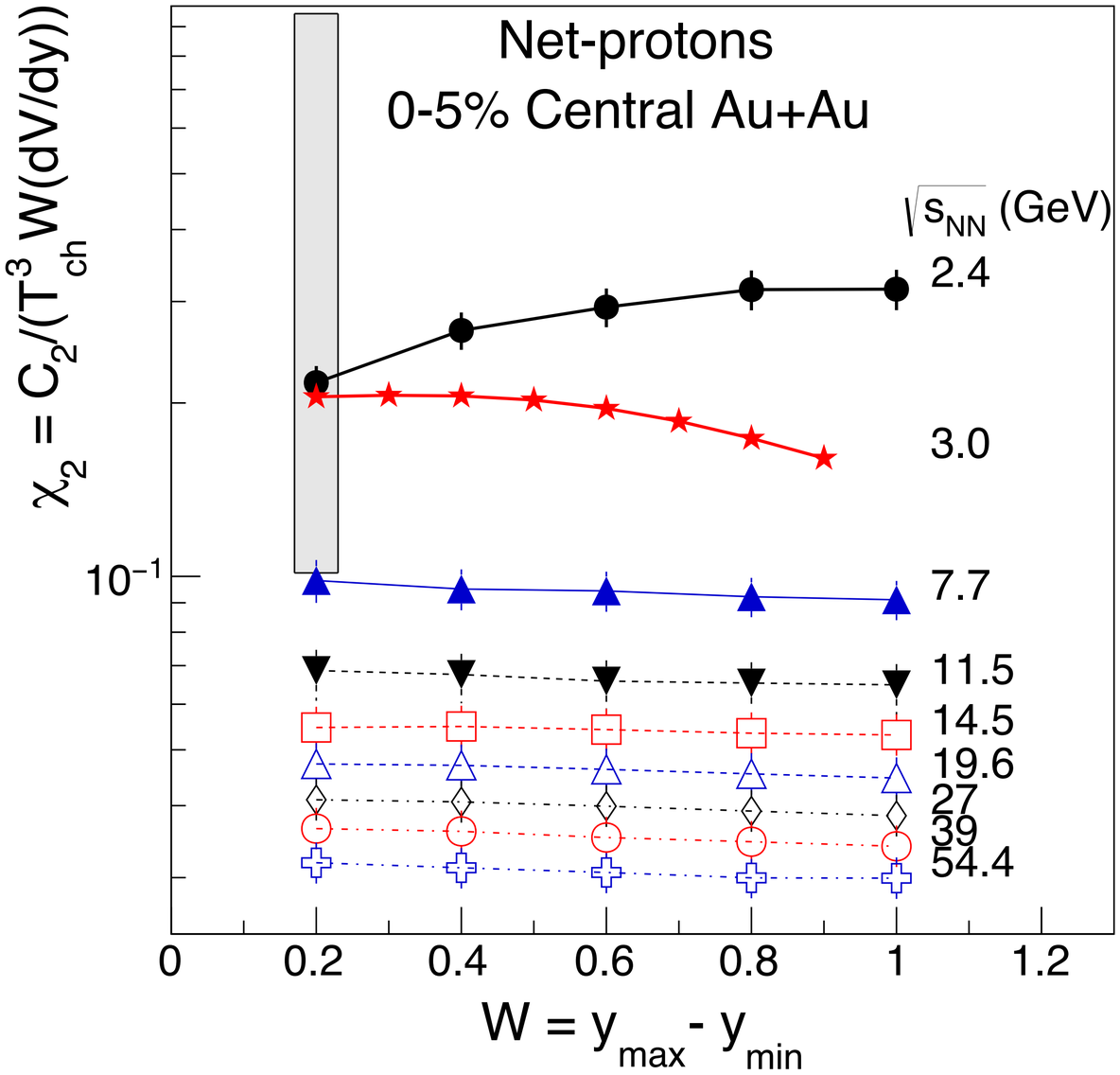
$\sqrt{s_{\mathrm{NN}}}$ (GeV)	y_{beam}	μ_{fo} (GeV)	T_{fo} (GeV)	dV_{fo}/dy (fm^3)
2.4	0.73	0.776	0.050	17157
3.0	1.05	0.720	0.080	4850
7.7	2.09	0.398	0.144	1044
11.5	2.50	0.287	0.149	1047
14.5	2.73	0.264	0.152	1080
19.6	3.04	0.188	0.154	1137
27	3.36	0.144	0.155	1218
39	3.73	0.103	0.156	1341
54.4	4.06	0.083	0.160	1487

J. Adamczewski-Musch et al. (HADES), Phys. Rev. C 102, 024914 (2020)
M. Abdallah *et al.* (STAR), Phys. Rev. C 104, 024902 (2021)
M. Abdallah *et al.* (STAR), Phys. Rev. C 107, 024908 (2023)
A. Andronic, P. Braun-Munzinger, J. Stachel, Acta Phys. Polon. B 40, 1005-1012 (2009)
A. Motornenko *et al.*, Phys. Lett. B 822, 136703 (2021)
S. Chatterjee *et al.*, Adv. High Energy Phys. 2015, 349013 (2015)

Susceptibility results

$$\chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$$

- Grey band shows uncertainty from freeze-out ambiguities for the 2.4 GeV data.
  Uncertainty precludes any conclusion about observing a maximum in χ_2
  (with a critical point, you should see a maximum in χ_2)

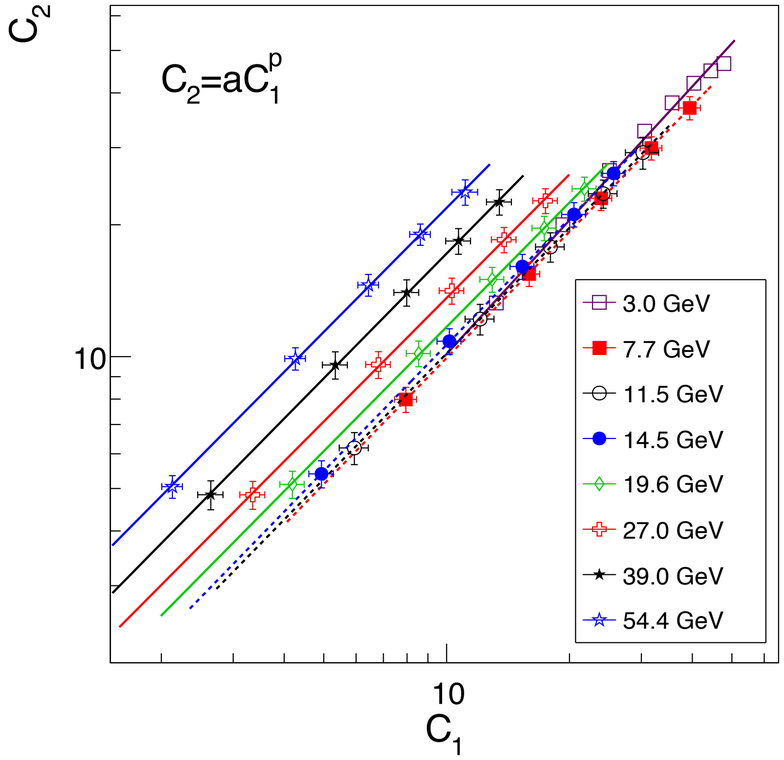


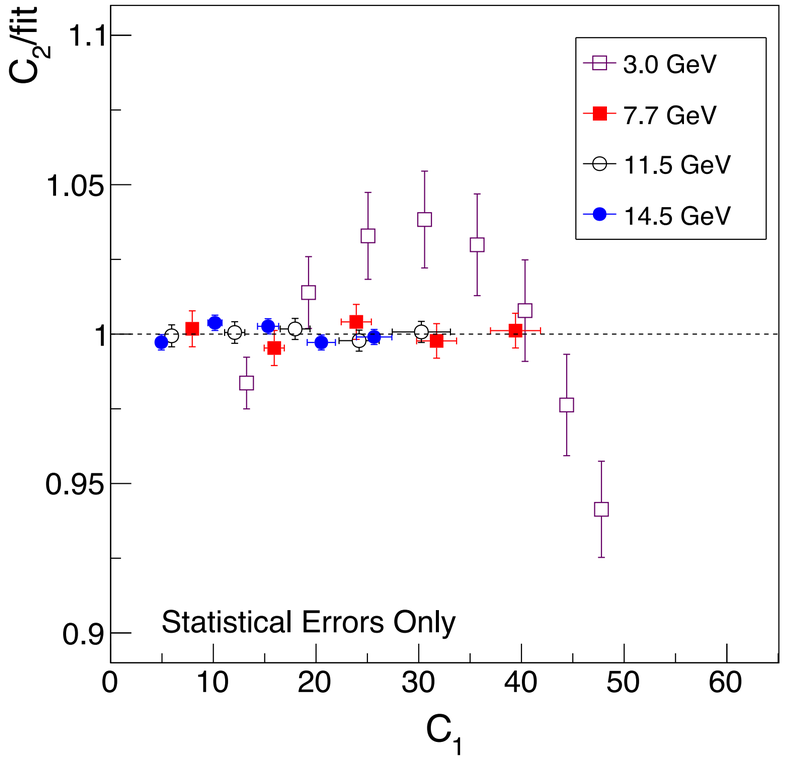
Where can we expect scaling behavior?

- For fluids far from the critical region, a mean-field treatment is good enough.
  The transition between the critical scaling region, intermediate scaling region, and extended scaling region has been studied: for fluids, the extended scaling region essentially covers the entire phase diagram where fluctuation contributions are small but finite.

  M.A. Anisimov, S.B. Kiselev, J.V. Sengers, S.Tang, Crossover approach to global critical phenomena in fluids, Physica A 188, 4 (1992)

- In the region of the phase diagram where the bulk of the evolution is well described by hydrodynamics (a scale free theory for $\eta \approx 0$), the data follows Taylor's Law: $\sigma^2 = a\lambda^p$ (scale free)





$C_2 = aW^p$

$C_2 = a(xW)^p = ax^pW^p = a'W^p$

where $C_1 \propto W$ in this energy range

Scale invariance supports the applicability of FSS for $\sqrt{s_{NN}} \geq 7.7$ GeV

What to scale with?

$$\xi_\infty(t,0) \sim |t|^{-\nu}$$

$$\xi_\infty(0,h) \sim |h|^{-\nu_c}$$

$$t \equiv \frac{T - T_c}{T_c}$$

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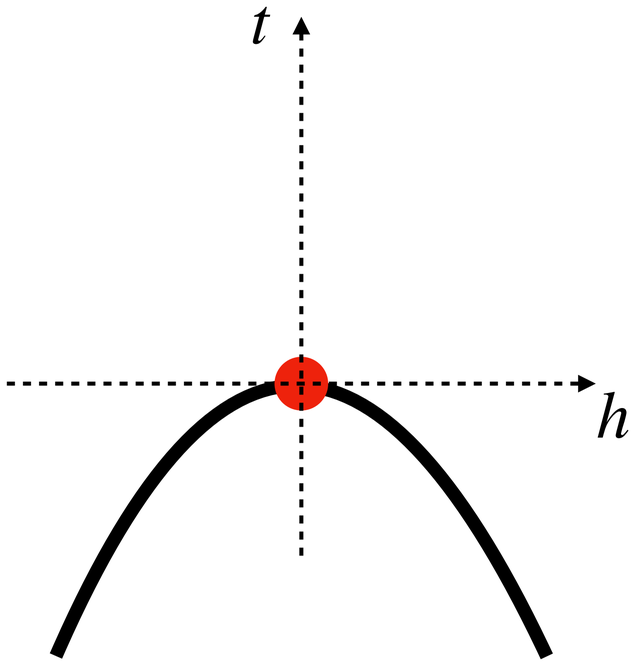

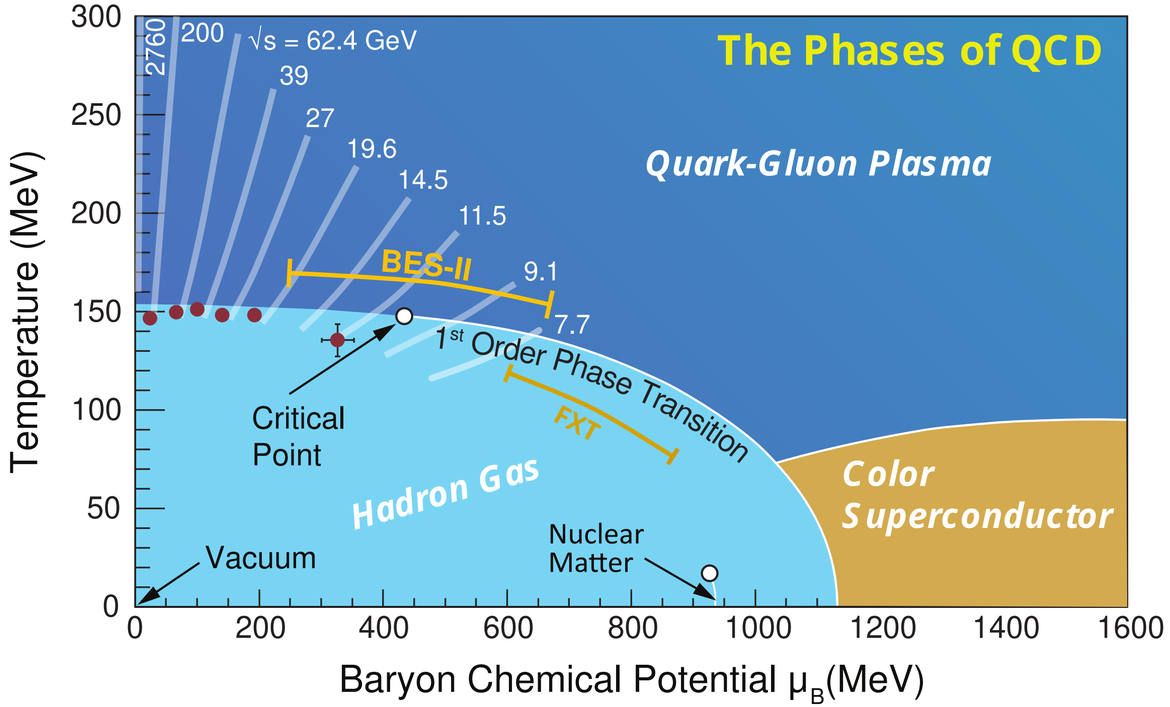

$$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

one can find CP by plotting

$$h(\mu, T) = -\frac{\cos\alpha_1 \Delta T + \sin\alpha_1 \Delta\mu}{wT_c \sin(\alpha_1 - \alpha_2)}$$

$$t(\mu, T) = \frac{\cos\alpha_2 \Delta T + \sin\alpha_2 \Delta\mu}{\rho w T_c \sin(\alpha_1 - \alpha_2)}$$

M.S. Pradeep, M. Stephanov, Phys. Rev. D **100** 5, 056003 (2019) arXiv:1905.13247

What to scale with?

We map ΔT and $\Delta\mu_B$ onto t and h:

- α_1 is determined from a straight line between the candidate CP $(\mu_{B,c}, T_c)$ and the $\sqrt{s_{NN}} = 7.7$ GeV freeze-out point
- For the other parameters, we take
  $\alpha_2 = \alpha_1 + \pi/2, \quad w = 1, \quad \text{and} \quad \rho = \mu_{B,c}/T_c$

$$h(\mu, T) = -\frac{\cos\alpha_1 \Delta T + \sin\alpha_1 \Delta\mu}{wT_c \sin(\alpha_1 - \alpha_2)}$$

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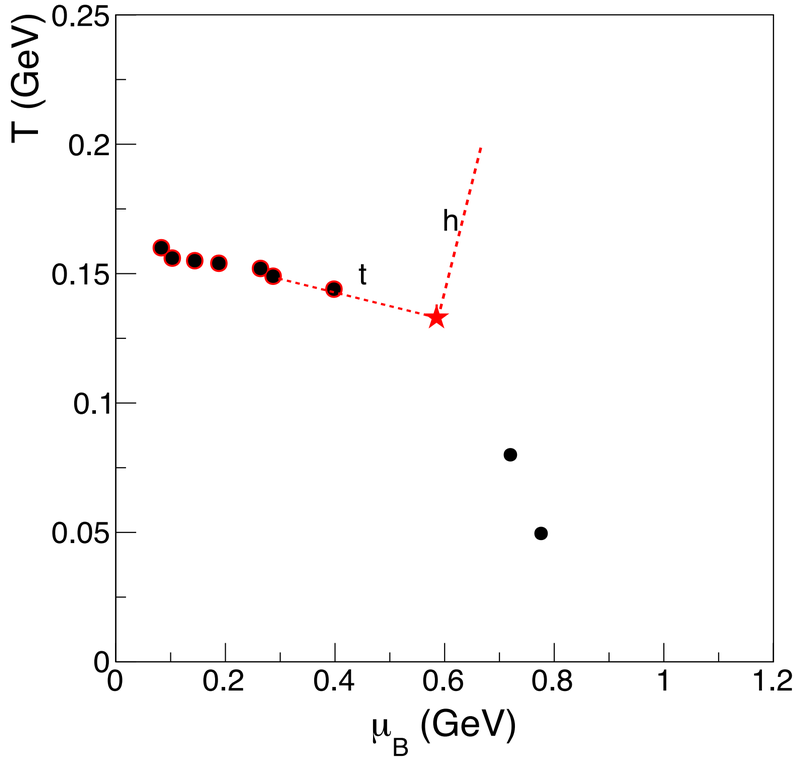

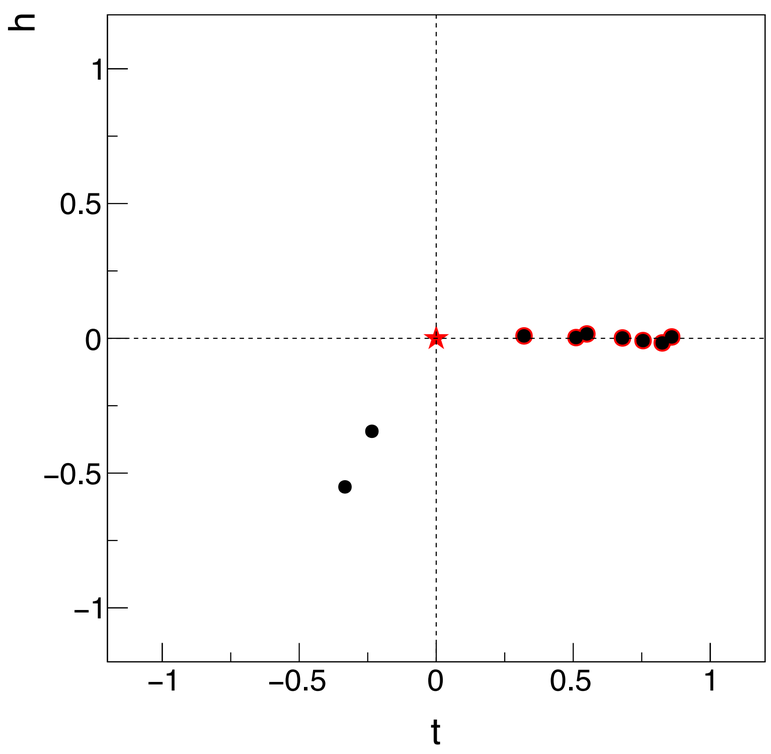

Scaled susceptibility: 2D fit w/ mean-field exponents

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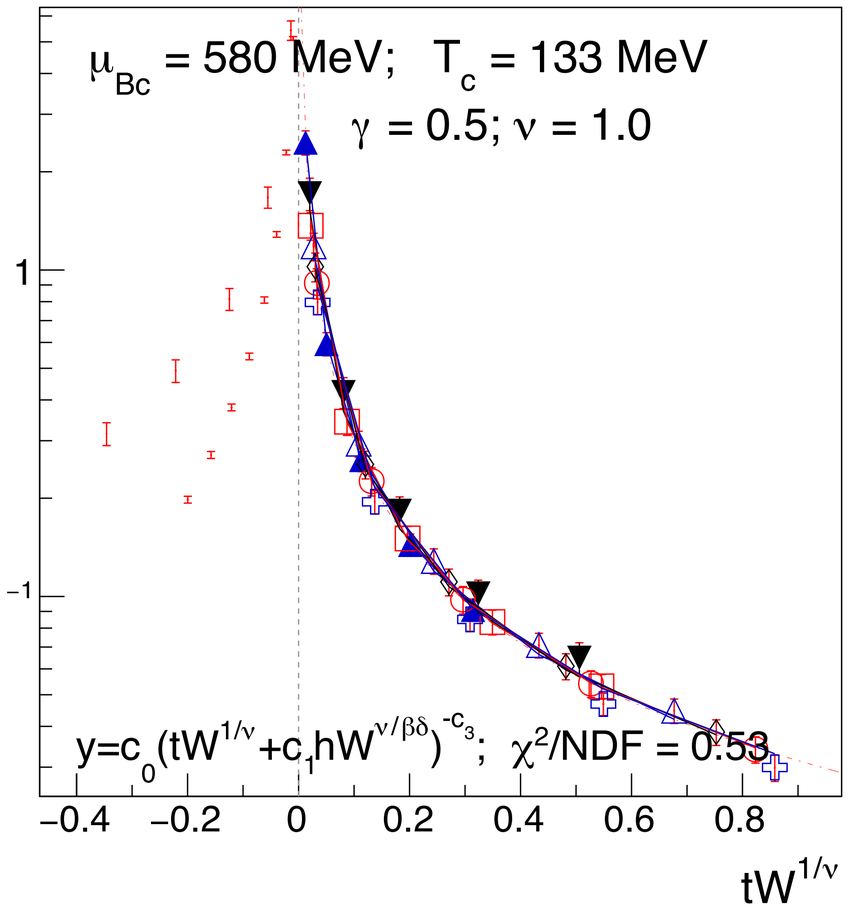
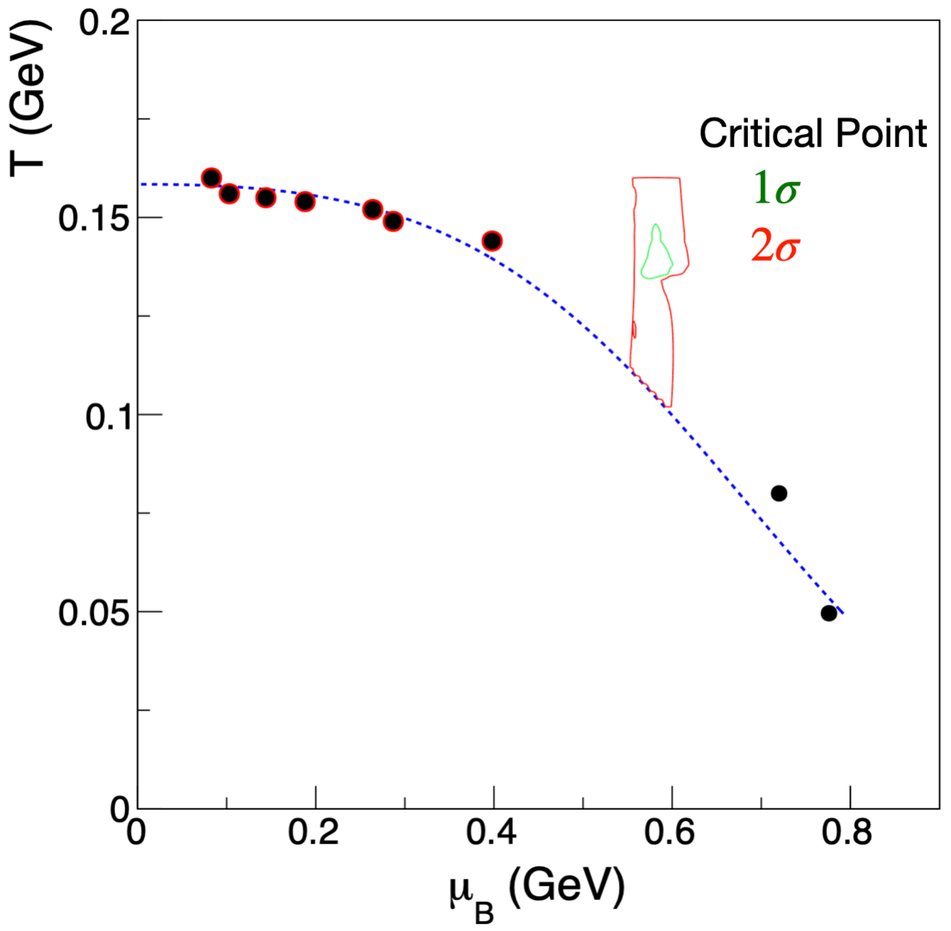
$$t \approx -\frac{\mu_B - \mu_{B,c}}{\mu_{B,c}}$$





With mean-field exponents, we find scaling for $555 < \mu_{B,c} < 610$ MeV;
T_c only constrained by "plausibility" (below $T_{pc,\mu_B=0}$ and above $T_{\rm fo}$)

Chi-square contours identify an allowed region in the phase diagram: $\mu_{B,c} = 580 \pm 30$ MeV

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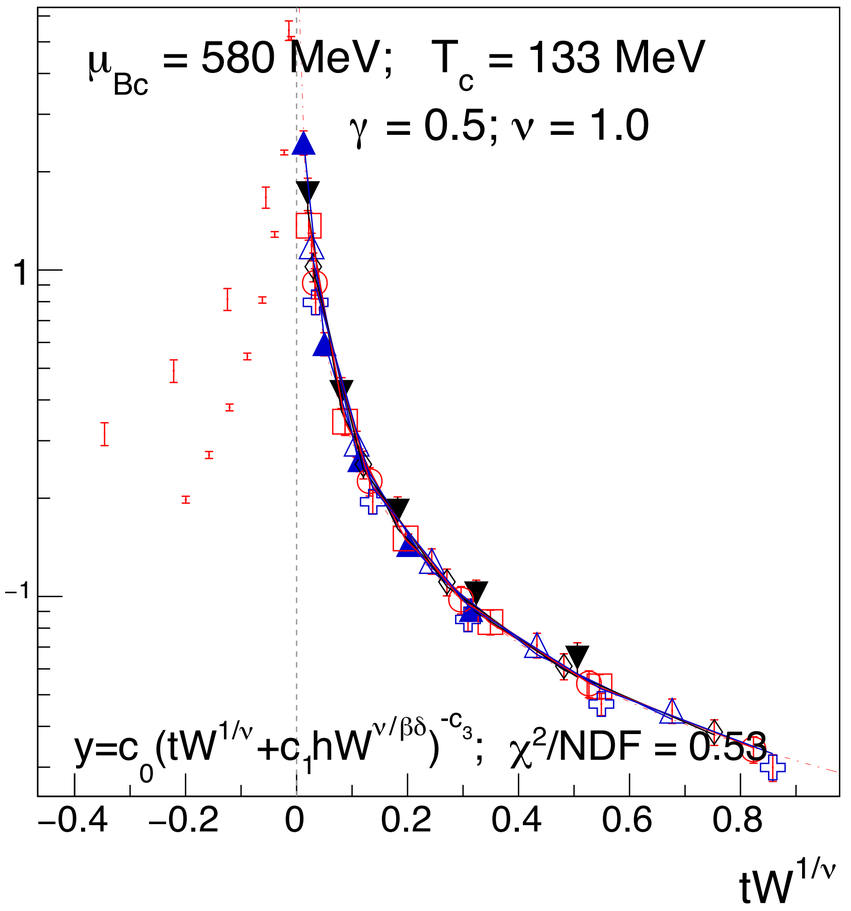

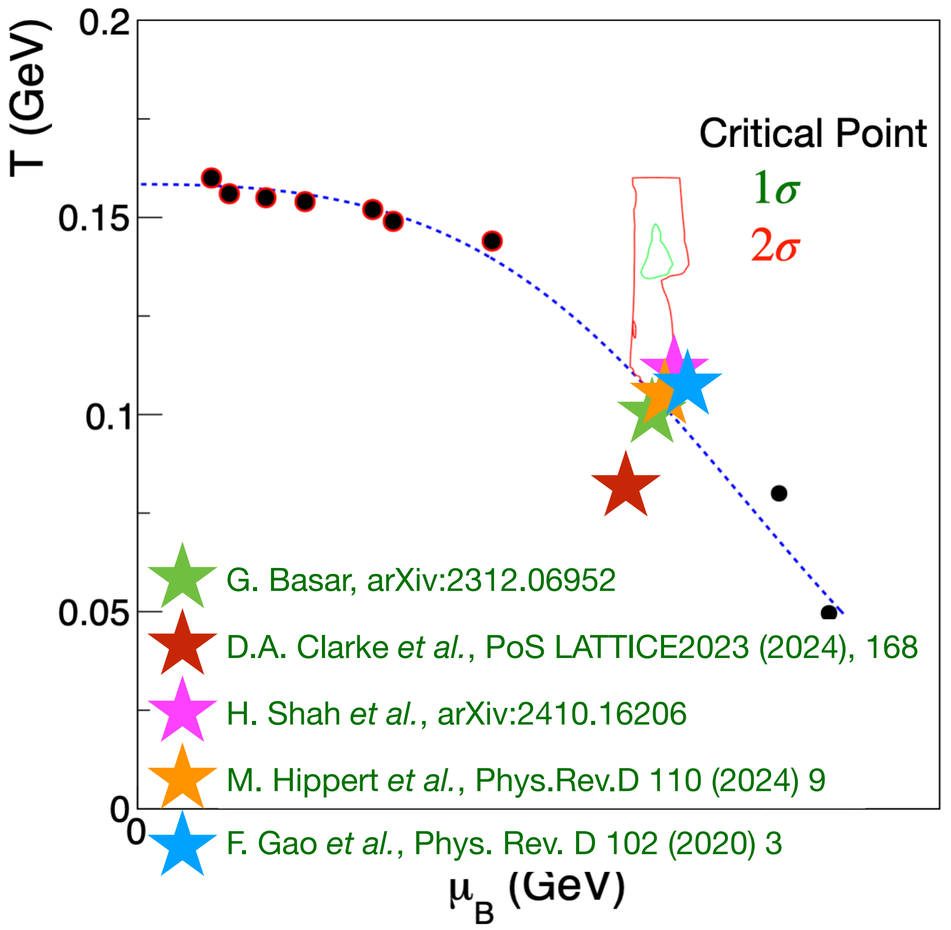

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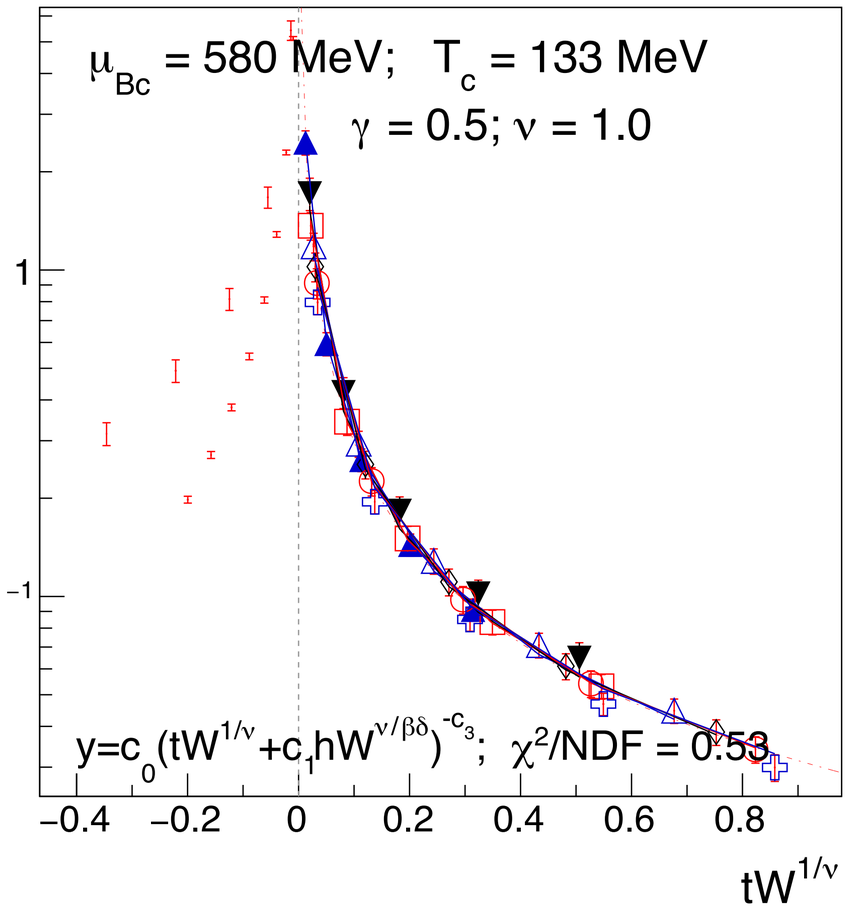

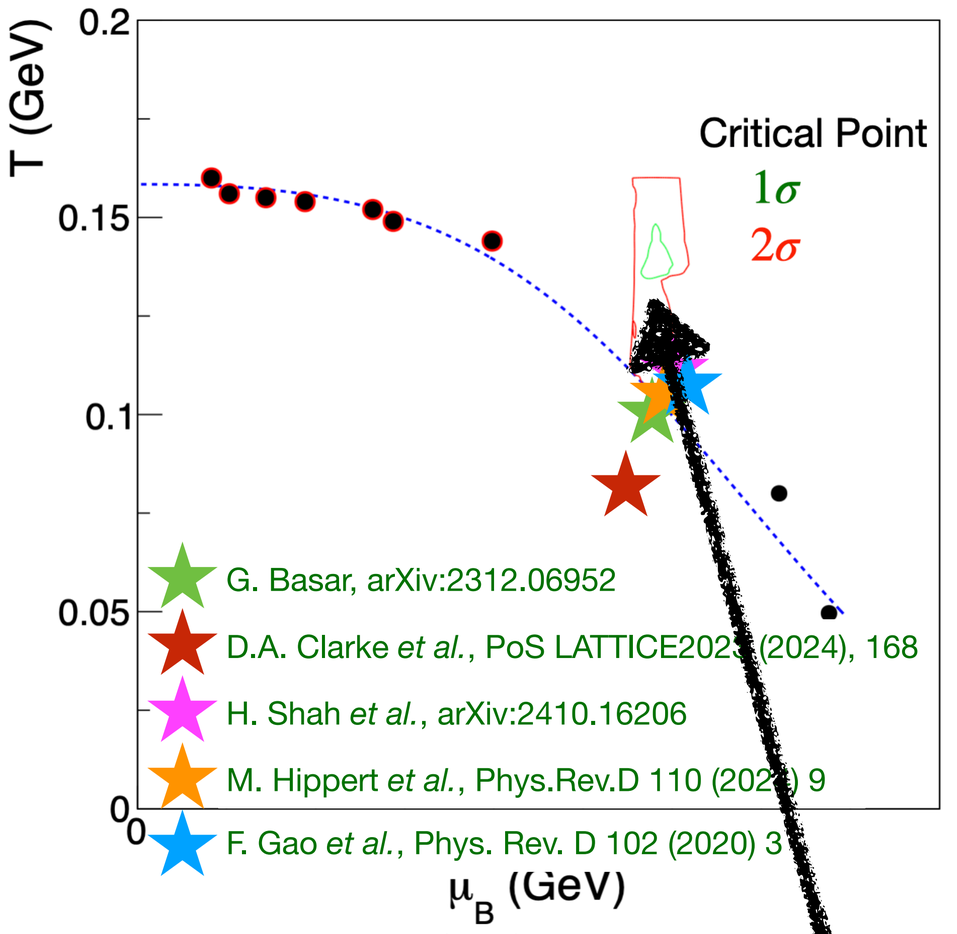

first extraction from experimental data!

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Chi-square contours identify an allowed region in the phase diagram: $\mu_{B,c} = 580 \pm 30$ MeV

Bonus: Binder cumulant

$$U_4 = -C_4/(3C_2^2)$$ K. Binder, Z. Phys. B 43, 119 (1981)

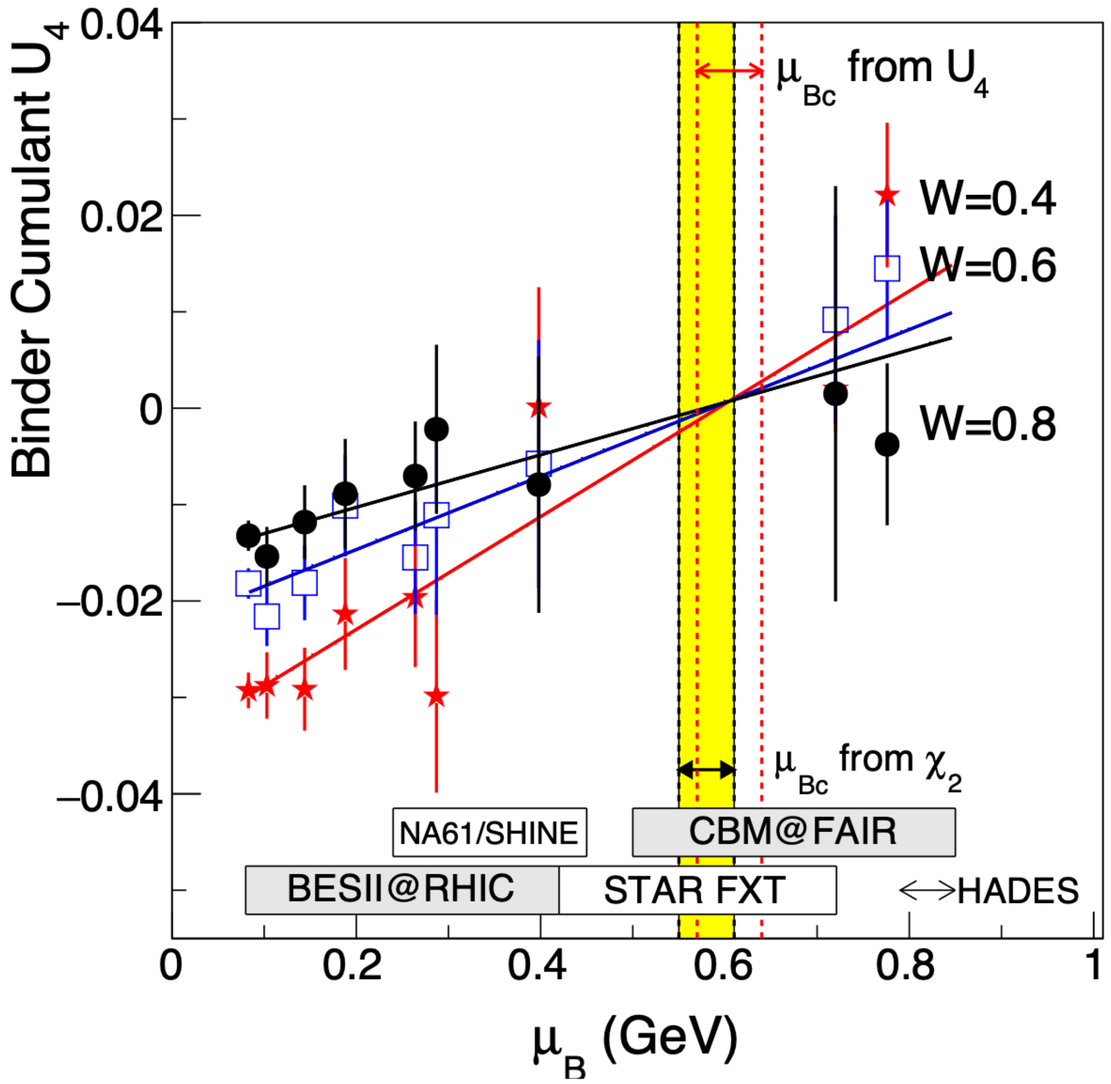
- Behavior of the Binder cumulant:
  Gaussian: $U_4 = 0$
  bimodal: $U_4 = 2/3$
  crosses at the critical point:
  $$U_4 \approx a_1 + a_2(\mu_B - \mu_{B,c})W^{1/\nu}$$

- At low μ_B, U_4 follows Skellam with
  $U_4(W = 0.8) > U_4(W = 0.6) > U_4(W = 0.4)$

- At $\mu_B > 400$ MeV, the ordering appears to reverse

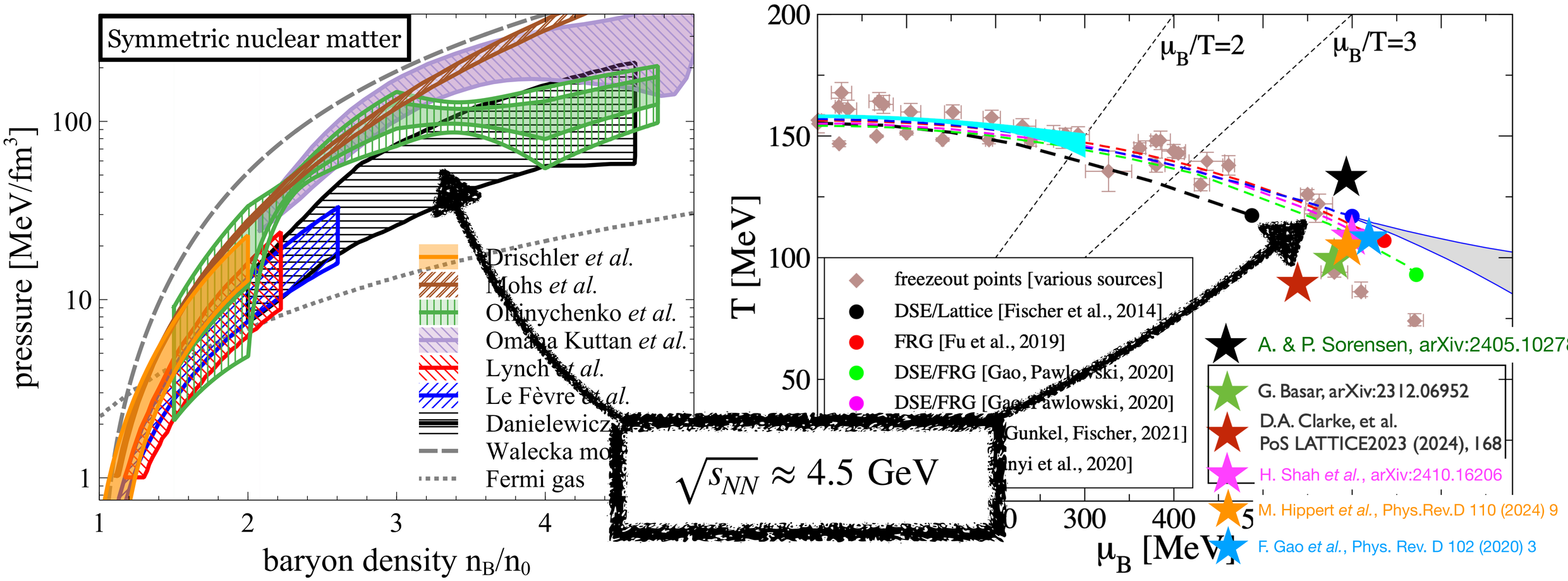
- Cross at $\mu_B = 605 \pm 35$ MeV, increases toward positive values

- Data are consistent with a critical point between μ_B of 550 and 640 MeV



Summary

- constraints on the n_B-dependence of the EOS: χEFT, transport model comparisons: old (3) and new (3)
- constraints on the QCD critical point: LQCD-based (3), holography+LQCD (1), STAR data-based (1)



adapted from Christian Fischer, CPOD 2024